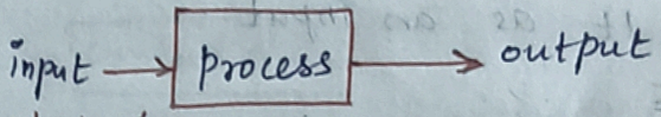
MODULE - 1

19/09/22 Monday

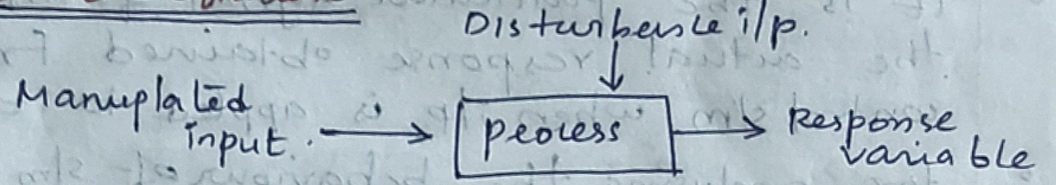
System: An inter connection of elements and devices for a desired Purpose.

control system: An inter connection of compoents forming a system configuration that will provide a desired response.

Process: The device, Plant, or s/m under control. The i/p and o/p relⁿship represents the cause and effect relⁿship of process.



Basic definitions



- **Plant**: The portion of a s/m which is to be controlled or regulated is called as the plant.
- A plant may be a peice of equipment, perhaps, Just a set of machine parts.
- The purpose of plant Is to perform a - particular operation

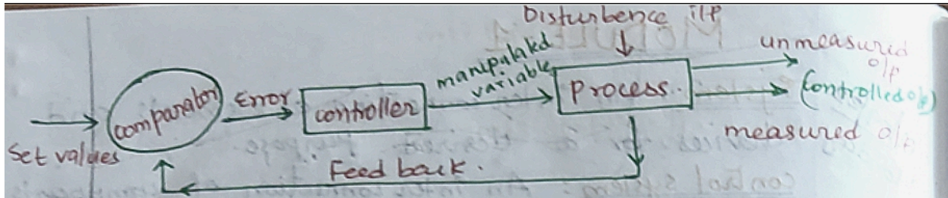
Eg:- mechanical device, a heating furnace, a chemical reactor/ spacradt

- **Process**:- Any operation to be controlled.

Eg:- chemical, economical, & biological process.

- **controller**: The element of the s/m itself or external to the s/m which controls the plant or the process. is called as controller.

Eg:- ON / OFF switch to control bulb.



Input (excitation)

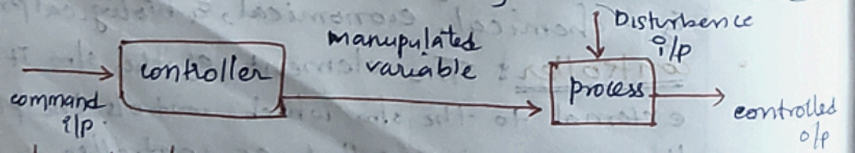
- It is an applied signal or an excitation signal applied to control s/m from an external energy source in order to produce a specified o/p
- for each s/m, there must be excitation and s/m accepts it as an input.

output

- It is the particular signal of interest of the actual response obtained from a control s/m when i/p is applied to it.
- for analysing the behaviour of s/m for such i/p, it is necessary to define the o/p of a s/m.

Disturbence

- It is a signal which tends to adversely affect the value of the o/p of a s/m.
- If such a disturbence is generated with in the s/m it self, it is called as internal disturbence.
- The disturbence generated outside the s/m acting as an extra i/p. to the s/m in addⁿ to the normal i/p affecting the o/p adversly is called as an external disturbence



→ classification of control s/m

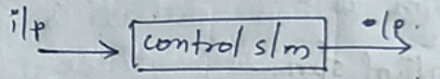
1) Natural control s/m → universe, → Human body

2) Man made control s/m → Vechicles, → Aeroplanes.

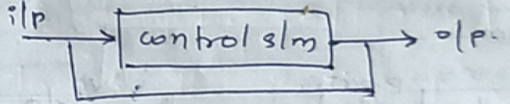
3) Manual control s/m → water level control, → Room temp. regulation via electronic fan.

4) Automatic control s/m → Human body temp control, → Room temp regl. via AC

5) open loop control s/m → washing Machine, → Toaster, → electronic fan.



6) closed loop control s/m → Refrigrator., → Drwierless car, → Auto-pilot s/m.



7) linear Vs Non-linear control s/m.

A control s/m in which o/p varies linearly with the i/p is called a linear control s/m.

8) Time invariant v/s Time variant

- when the characteristics of the s/m do not depend up on time it self then the s/m is said to time invariant control s/m.
- Time varying control s/m is a s/m in which one or more parameters vary with time.

9) continous Data vs Discrete data s/m

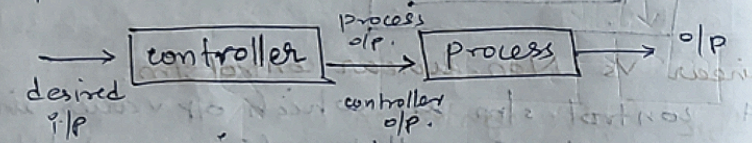
- In continous data control s/m all s/m variable as function of a continous time t.
- A DT control s/m involves one or more variable that are known only at DT intervals.

10) Deterministic vs stochastic control s/m.

A control s/m is deterministic if the response to i/p is predictable and repeatable.
If not control s/m is a stochastic control s/m

→ open loop control s/m.

- Any physical s/m which does not automatically correct for variation in its o/p, is called an open-loop s/m.



- accuracy is very less.

Advantages.

- simple in construction
- Very much convenient when o/p is difficult to measure
- ease of maintenance.
- generally these are not troubled with the problems of stability
- such s/m are simple to design and hence economical

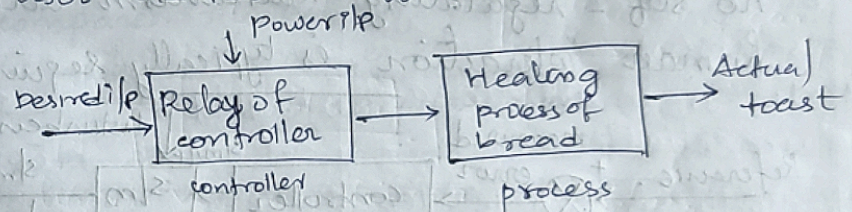
dis Advantage.

Accuracy is less
calibration

Eg. of an open loop s/m.

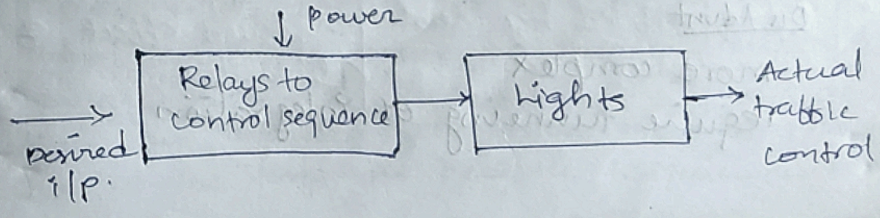
1) Automatic Toaster s/m.

- In this s/m, the quality of toast depends up on the time for which the toast is heated.
- Depending on the time setting, bread is simply heated in this s/m.
- The toast quality is to be judged by the user and has no effect on the inputs.



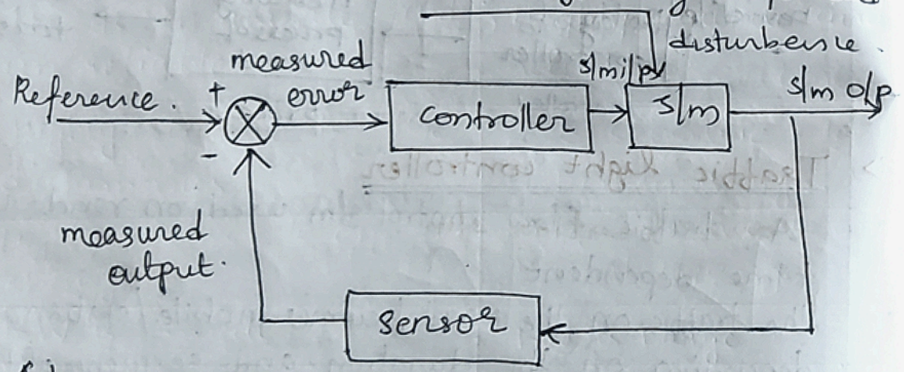
2) Traffic light controller

- A traffic flow control s/m used on roads is time dependent
- The traffic on the road becomes mobile/stationary depending on the duration and sequence of lamp glow.
- The sequence and duration are controlled by relays which are predetermined and not dependent on the rush on the road.



closed loop control system

A closed loop control s/m is a mechanical or electronic device that automatically regulates a system to maintain a desired state or set point without human interaction. It uses a feed back s/m. or sensor. closed loop control is contrasted with open loop control, where there is no self - regulating mechanism and human interaction is typically required



Advnt:

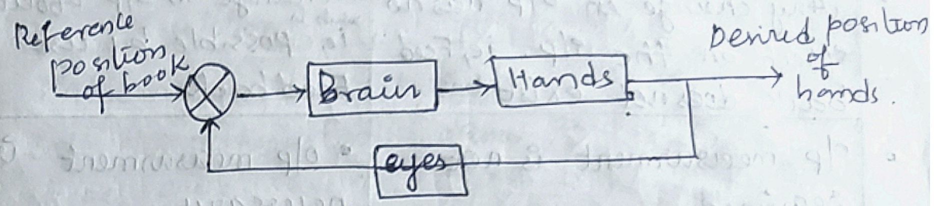
- more resource efficient
- more reliable and stable output.
- can control for external factor

Dis Advnt

- more complex
- require tunneing or integration

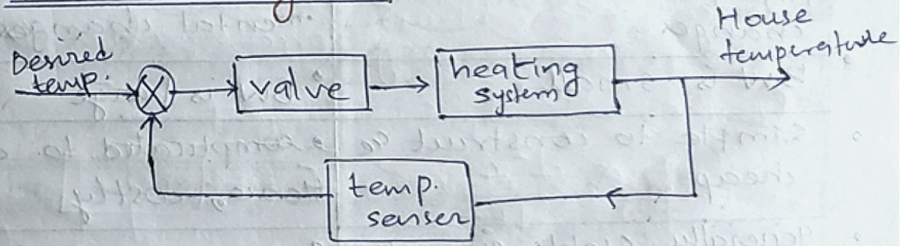
Eg. of closed loop control s/m.

1. **Human being**



If a person wants to reach for a book on the table. the position of the book is given as reference. Feed back signals from eyes compares the actual position of hand with reference position. error signal is given to brain. brain manipulate this error and gives signals to hands. This process contenues till the position of the hands get acheved that book.

2. **Home heating s/m.**

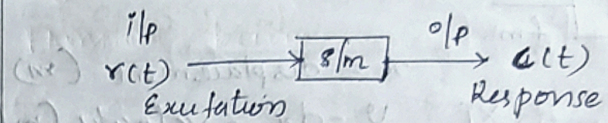


Here, heating s/m is operated by a valve the actual temp is sensed by a thermal senser. & compared with the desired temp. The d/b b/w these two actuates the valve mechanism to change the temp as per the requirement

Differene b/w open loop s/m & closed loop s/m

open loop s/m	closed loop s/m
• Any change in o/p has no effect on the i/p. ie, Feed back does not exist.	• change in o/p affects the i/p which is ie, possible by use of Feed back.
• o/p measurment is not required.	• o/p measurment is necessary
• Feed back element is absent.	• Feed back element is Present
• Error detetor (comparator) is absent	• comparator is present.
• It Is in aurate and un reliable	• Highly aurate and reliable
• Highly sensitive to the disturbences	• Less sensitive to the disturbences
• Highly senstive to enviornmental changes.	• Less senstive to enviornmental changes
• BW is small	• BW is large
• simple to construct & cheap	• complicated to design & Hence, costly
• generally, stable in nature	• It is un-stable
• Highly affeted by non-linearities	reduced effect of non-linearities

Mathematical model of a control s/m.



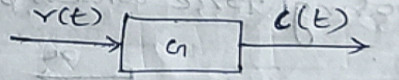
Mathematical model of a c.s consist of set of defferential eqns. and. The different types are:

1) differential equs.
2) Transfer functions $= \frac{o/p(s)}{i/p(s)}$ (zero intial condn.
3) SFG (signal flow graph)
4) Block diagram.

Transfer function

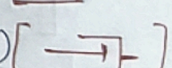
It is the ratio of L.T of o/p to the L.T of i/p under zero intial condition

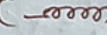
$$\text{Transfer function} = \frac{C(s)}{R(s)}$$



Mechanical s/m

Basic elements are: Mass (M) $\boxed{M}$

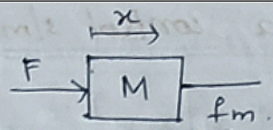
dash pot (B) []

() Spring (K)

'K' is the stiffness of a spring.

'B' is the Viscous friction coefficient. (N-sec/m)

Here, we use newton IInd law of motion to convert a mechanical s/m in to it's mathematical form



x = displacement (m)

$v = \frac{dx}{dt}$ = velocity (m/s)

$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ = accelration (m/s²)

$$f_m = M\frac{d^2x}{dt^2}$$

f = Appliedforce N (newton)

f_m = opposing force offered by mass of the body, N

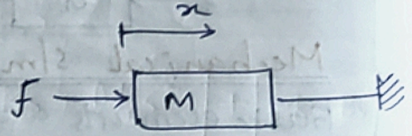
f_k - opposing force offered by the elasticity of the body (spring), N.

f_b - opposing force offered by the friction of the body (dash-pot), N.

FORCE BALANCE EQN. OF IDEALIZED ELEMENTS

consider an ideal mass element which has neligible friction and elasticity let a force be applied on it. The mass will offer an opposing force which is ∝ accel^rn of the body.

$f_m \propto \frac{d^2x}{dt^2}$.



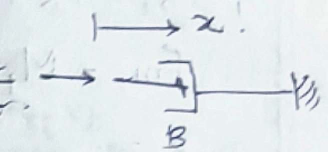
Ideal mass elements

$f_m = M\frac{d^2x}{dt^2}$.

$$f = f_m = M\frac{d^2x}{dt^2}.$$

→ Let a force be applied the dash pot will offer an opposing force which is ∝ velocity of the body

$f_b \propto \frac{dx}{dt}$ or $f = B\frac{dx}{dt}$.



~~representation~~ Ideal dashpot with one end fixed to reference.

By Newton II^nd law,

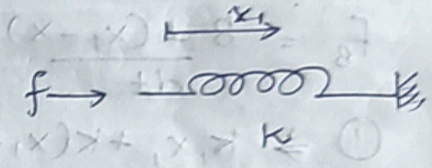
$$f = f_b = B\frac{dx}{dt}$$

spring

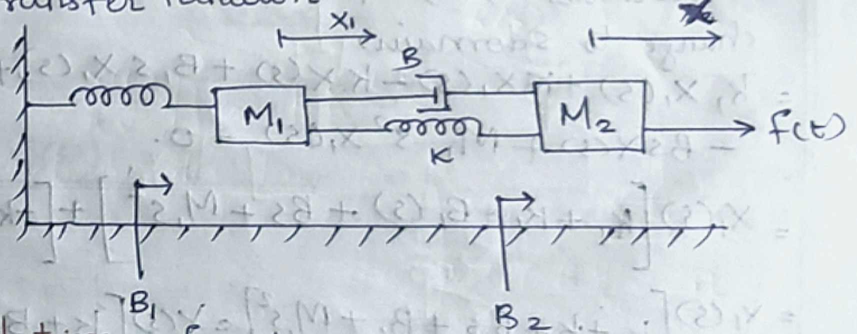
$f_k \propto x$.

$f_k = kx$

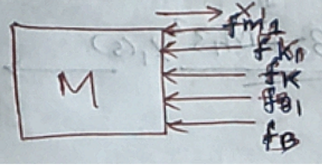
$$f = f_k = kx$$



Q1: write the DE governing the mechanical s/m shown in figure. & Determine the Transfer function



solution Free body diagram of M_1



by newton II^nd law,

opposing force $= f_{k_1} + f_k + f_{B_1} + f_B$ → ①

Applied force = 0

$f_{m_1} + f_{K_1} + f_K + f_{B_1} + f_B = 0 \longrightarrow ①$

$f_{m_1} = M_1 \frac{d^2 x_1}{dt^2}$

$f_{K_1} = K_1 x_1$

$f_K = K(x_1 - x)$

$f_{B_1} = B_1 \frac{dx_1}{dt}$

$f_B = B \frac{d(x_1 - x)}{dt}$

$① \Rightarrow K_1 x_1 + K(x_1 - x) + B_1 \frac{dx_1}{dt} + B \frac{d}{dt}(x_1 - x)$

(Time domain) $+ M \frac{d^2 x_1}{dt^2} = 0$

Taking Laplace Transform of above eqⁿ

Variables → x, x₁ and t

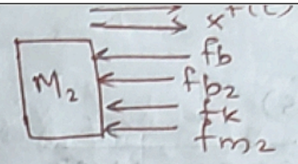
change to s domains

$= K_1 X_1(s) + K X_1(s) - K X(s) + B_1 s X_1(s) + B s X_1(s)$
$- Bs X(s) + M_1 s^2 X_1(s) = 0.$

$= X_1(s)\left[K_1 + K + B_1(s) + Bs + M_1 s^2\right] + \left[-K - Bs\right] X(s) = 0$

$= X_1(s)\left[K_1 + K + B_1 s + B_s + M_1 s^2\right] = X(s)\left[K + Bs\right]$

$X(s) = \frac{K_1 + K + B_1 s + B_s + M_1 s^2}{K + Bs} X_1(s) \longrightarrow ②$



$f_b = B \frac{d}{dt}(x - x_1)$

$f_{b_2} = B_2 \frac{dx}{dt}$

$f_K = K(x - x_1)$

$f_{m_2} = M_2 \cdot \frac{d^2 x}{dt^2}$

$f(t) = f_b + f_{b_2} + f_K + f_{m_2}$.

$f(t) = B \frac{d}{dt}(x - x_1) + B_2 \frac{d}{dt} x + K(x - x_1) + M_2 \frac{d^2 x}{dt^2}$

Taking Laplace Transforms

$F(s) = Bs X(s) - Bs X_1(s) + B_2 s X(s) + K X(s) - K X_1(s)$
$+ M_2 s^2 X(s)$

$= X(s)\left[Bs + B_2 s + K + M_2 s^2\right] - X_1(s)\left[Bs + K\right] \longrightarrow ③$.

② in above eqⁿ.

$F(s) = \left(\frac{K_1 + K + B_1 s + Bs + M_1 s^2}{K + Bs}\right) X_1(s) \cdot \times \begin{bmatrix} Bs + B_2 s + K + \\ M_2 s^2 \end{bmatrix}$
$- X_1(s)\left[Bs + K\right]$

$= X_1(s) \Big\{$

$TF = \frac{L(o/p)}{L(i/p)}$

given s/m $o/p = X(s) \ (t - x)$

$i/p = F(s) \ (t - f(t))$

$TF = \frac{X(s)}{}$

from eqⁿ ②

$$X_1(s) = \frac{X(s)\,(K+Bs)}{K_1+K+B_1s+Bs+M_1s^2}$$

sub $X_1(s)$ in eqⁿ ③,

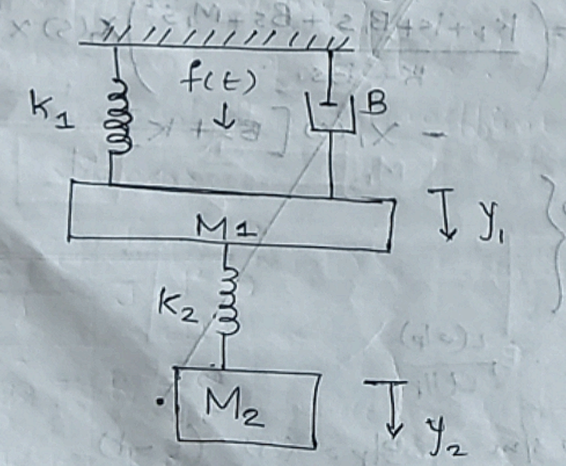
$$F(s) = (Bs+B_2s+K+M_2s^2)\,X(s) - \frac{(K+Bs)^2 X(s)}{K_1+K+B_1s+Bs+M_1s^2}$$

(Take L.C.M)

$$\left\{\frac{(Bs+B_2s+K+M_2s^2)(K_1+K+B_1s+Bs+M_1s^2)-(K+Bs)^2}{K_1+K+B_1s+Bs+M_1s^2}\right\}X(s)$$

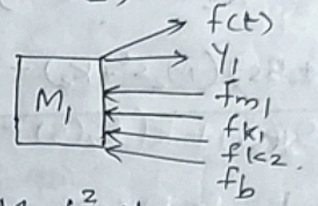
$$TF = \frac{X(s)}{F(s)} = \frac{K_1+K+B_1s+Bs+M_1s^2}{(Bs+B_2s+K+M_2s^2)(K_1+K+B_1s+Bs+M_1s^2)-(K+Bs)^2}$$

2. Determine the transfer function $\frac{Y_2(s)}{F(s)}$ of the system shown in figure.



Solⁿ. Transfer function $= \frac{Y_2(s)}{F(s)}$.

consider M_1,



By newtons law,

$f(t) = f_{m_1} + f_{k_1} + f_{k_2} + f_b$

$$f(t) = M_1 \frac{d^2}{dt^2} y_1 + K_1 y + K_2(y - y_2) + B\frac{d}{dt}y_1 \longrightarrow ①$$

Taking L.T.

$$F(s) = M_1 s^2 Y_1(s) + K_1 Y_1(s) + K_2(Y_1(s) - Y_2(s)) + B\,s\,Y_1(s)$$

$$F(s) = (M_1 s^2 + K_1 + K_2 + Bs)\,Y_1(s) - K_2 Y_2(s) \longrightarrow ②$$

consider M_2,

$f_{m_2} + f_{k_2} = 0$

$$M_2 \frac{d^2 Y_2}{dt^2} + K_2 (Y_2 - Y_1) = 0 \longrightarrow ③$$

Applying L.T.

$$M_2 s^2 Y_2(s) + K_2 Y_2(s) - K_2 Y_1(s) = 0 \longrightarrow ④$$

$$T.F = \frac{Y_2(s)}{F(s)}$$

from ④, $(M_2 s^2 + K_2)\,Y_2(s) = K_2 Y_1(s)$

$$Y_1(s) = \frac{M_2 s^2 + K_2\,Y_2(s)}{K_2} \longrightarrow ⑤$$

sub eq^n ⑤ in ② then,

$$F(s) = \left\{ (M_1 s^2 + K_1 + K_2 + Bs) \frac{(M_2 s^2 + K_2)}{K_2} - K_2 \right\} Y_2(s)$$

(LCM)

$$F(s) = \left\{ \frac{(M_1 s^2 + K_1 + K_2 + Bs)(M_2 s^2 + K_2) - K_2^2}{K_2} \right\} Y_2(s)$$

$$T.F = \frac{Y_2(s)}{X_2(s)} = \frac{K_2}{(M_1 s^2 + K_1 + K_2 + Bs)(M_2 s^2 + K_2) - K_2^2}$$

Mechanical rotational s/m

θ = angular displaucmt, rad.

$\frac{d\theta}{dt}$ = angular velouty, rad/sec.

$\frac{d^2\theta}{dt}$ = angular acl^n. (α)

T = applied Torque.

J = moment of inertia.

B = rotational frictional coeffint

K = stiffness

linear	Rotat
x	θ
v	ω
a	α
B	B
K	K
M	J
F	τ

opposing forces;

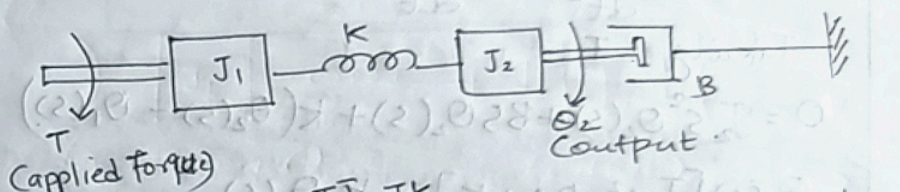
$$f_J = J \frac{d^2\theta}{dt^2}$$

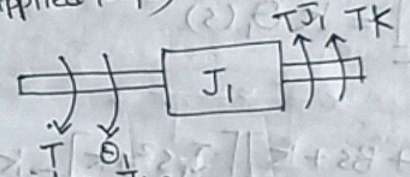
$$f_b = B \frac{d\theta}{dt}$$

$$f_K = K\theta$$

Q) write the differential eq^n governing the mechanical rotational s/m shown in fig.

1) obtain the transfer function of the s/m.





θ₁ → displacement

consider J₁, $T_{J_1} = fm_1$ (force due to mass) Here angular,

$$J_1 \cdot \frac{d^2\theta_1}{dt^2}$$

$$T_K = K(\theta_1 - \theta_2)$$

$$T = T_{J_1} + T_K$$

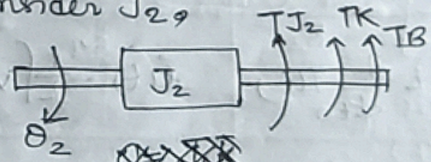
$$T = J \cdot \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta_2)$$

Taking L.T.

$$T(s) = J_1 s^2 \theta_1(s) + K(\theta_1(s) - \theta_2(s))$$

$$= \theta_1(s)\left[J_1 s^2 + K\right] - K\theta_2(s) \longrightarrow ①$$

consider J₂,



Torque is not here put = 0.

$$T_{J_2} = J_2 \frac{d^2\theta_2}{dt^2}$$

$$T_K = K(\theta_2 - \theta_1)$$

$$T_B = B \frac{d\theta_2}{dt}$$

$0 = T_{J_2} + T_B + T_K$

$$0 = J_2 \frac{d^2\theta_2}{dt^2} + B\frac{d\theta_2}{dt} + K(\theta_2 - \theta_1)$$

Taking L.T.

$$0 = J_2 s^2 \theta_2(s) + Bs\theta_2(s) + K(\theta_2(s) - \theta_1(s))$$

$$\frac{\theta_2(s)[J_2 s^2 + Bs + K]}{K} = K\theta_1(s)$$

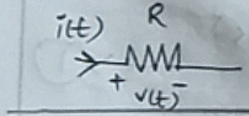
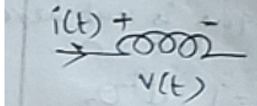
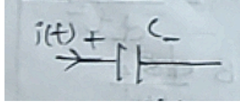
$$T(s) = \frac{\theta_2(s)[J_2 s^2 + Bs + K]}{K}[J_1 s^2 + K] - K\theta_2(s)$$

crossmultiply

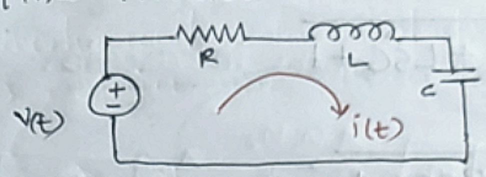
$$T(s) = \frac{\theta_2(s)[[J_2 s^2 + Bs + K][J_1 s^2 + K] - K^2]}{K}$$

$$\frac{\theta_2(s)}{T(s)} = \frac{K}{[[J_2 s^2 + Bs + K][J_1 s^2 + K] - K^2]}$$

Electrical Systems.

Element	Vtg acrosselement.	I through the element
	$v(t) = R\,i(t)$	$i(t) = \frac{v(t)}{R}$
	$v(t) = L\frac{d}{dt}i(t)$	$i(t) = \frac{1}{L}\int v(t)dt$
	$v(t) = \frac{1}{C}\int i(t)dt$	$i(t) = C\frac{dv(t)}{dt}$

Q) find Transfer function of RLC series circuit?



L T $\frac{d}{dt} = s$

L T $\int = \frac{1}{s}$

L T $\frac{d^2}{dt^2} = s^2$

input → V(t)

output → V₀(t)

$$T.F = \frac{V_0(s)}{V(s)}$$

Applying KVL in loop or circuit

$$V(t) - Ri(t) - L\frac{d}{dt}i(t) - \frac{1}{c}\int i(t)dt = 0.$$

Taking Laplace Transform,

$$V(s) - RI(s) - LsI(s) - \frac{1}{c}\times\frac{1}{s}I(s) = 0$$

$$V(s) - \left(R + Ls + \frac{1}{Cs}\right)I(s) = 0$$

or

$$V(s) = \left\{\frac{RCs + LCs^2 + 1}{Cs}\right\}I(s) \longrightarrow ①$$

$V_0(t)$ = Voltage across capacitor

$$V_0(t) = \frac{1}{c}\int i(t)dt$$

Taking Laplace Transforms

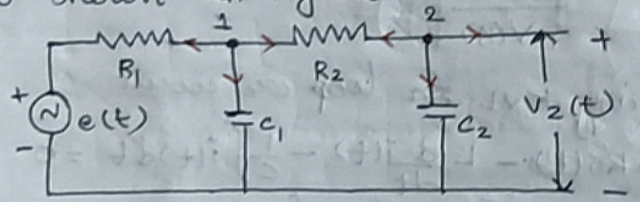
$$V_0(s) = \frac{1}{Cs}I(s) \longrightarrow ②$$

$$t.F = \frac{②}{①} \qquad TF = \frac{V_0(s)}{V(s)} = \frac{\frac{1}{Cs}I(s)}{\frac{RCs + LCs^2 + 1}{Cs}I(s)}$$

$$= \frac{1}{CS} \times \frac{CS}{RCS + LS^2C + 1}$$

$$T.F = \frac{1}{LCS^2 + RCS + 1}$$

Q2) Obtain the transfer function of the electrical n/w shown in figure.



sol^n By nodal analysis, voltage across node 1 is V_1 and across node 2 ie V_2
(all current are leaving concept)

By using KCL @ node 1,

$$\frac{V - e(t)}{R_1} + C\frac{d}{dt}V_1(t) + \frac{V_1 - V_2}{R_2} = 0.$$

Taking Laplace Transform.

$$\frac{V_1(S) - E(S)}{R_1} + CS\,V_1(S) + \frac{V_1(S) - V_2(S)}{R_2} = 0.$$

$$V_1(S)\left[\frac{1}{R_1} + CS + \frac{1}{R_2}\right] - \frac{1}{R_2}V_2(S) = \frac{E(S)}{R_1} \longrightarrow ①$$

Apply KCL @ node 2,

$$\frac{V_2 - V_1}{R_2} + C_2\frac{d\,V_2(t)}{dt} = 0$$

apply Laplace Transform.

$$\frac{V_2(S) - V_1(S)}{R_2} + C_2 S\,V_2(S) = 0.$$

$$V_2(S)\left[\frac{1}{R_2} + C_2 S\right] = \frac{V_1(S)}{R_2}$$

$$V_1(S) = V_2(S)\left[\frac{1}{R_2} + C_2 S\right] \times R_2$$

$$V_1(S) = \left[1 + R_2C_2S\right]V_2(S) \longrightarrow ②$$

② in ①,

$$\left[1 + R_2C_2S\right]\left[\frac{1}{R_1} + C_1S + \frac{1}{R_2}\right]V_2(S) - \frac{1}{R_2}V_2(S) = \frac{E(S)}{R_1}.$$

$$\left\{(1 + R_2C_2S)\left(\frac{R_2 + C_1SR_1R_2 + R_1}{R_1R_2}\right) - \frac{1}{R_2}\right\}V_2(S) = \frac{E(S)}{R_1}.$$

$$\frac{(1 + R_2C_2S)(R_2 + R_1R_2C_1S + R_1) - R_1}{R_1R_2} = \frac{E(S)}{R_1V_2(S)}$$

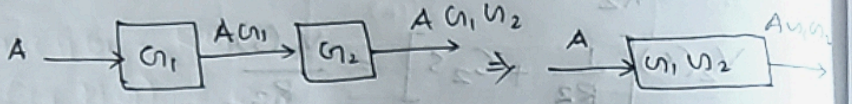
$$T.F = \frac{V_2(S)}{E(S)}$$

$$TF = \frac{1}{\cancel{R_1}}\left\{\frac{\cancel{R_1}\;R_2}{(1 + R_2C_2S)(R_2 + R_1R_2C_1S + R_1) - R_1}\right\}$$

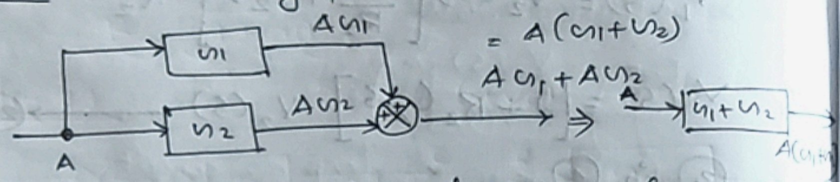
$$T.F = \frac{R_2}{(1 + R_2C_2S)(R_2 + R_1R_2C_1S + R_1) - R_1}$$

BLOCK DIAGRAM REDUCTION.

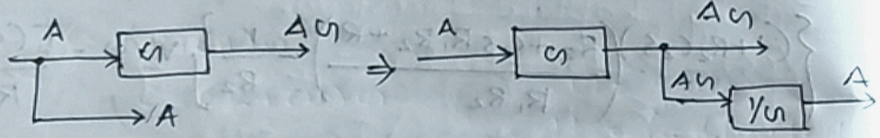
Rule-1: combining the blocks in cascade.



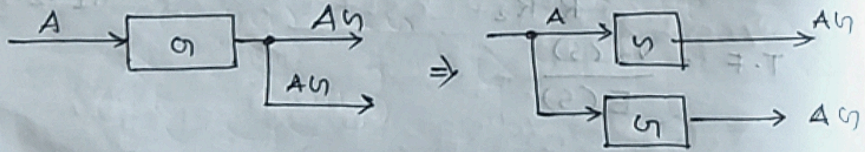
Rule 2: combining parallel blocks.



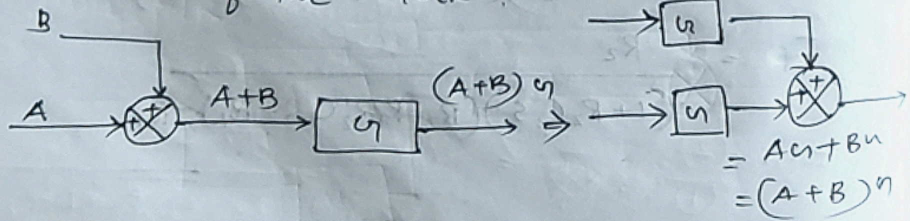
Rule-3: moving the branch point ahead of the block.



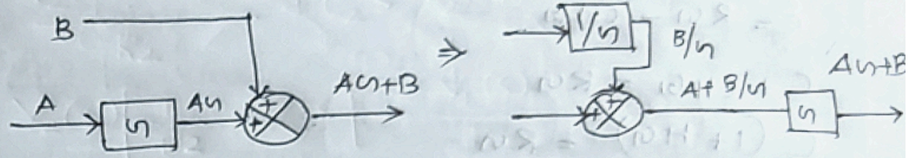
Rule 4: moving the branch point before the block



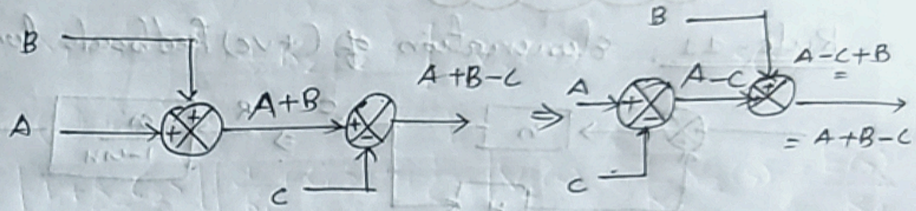
Rule-5: moving the summing point ahead of the block.



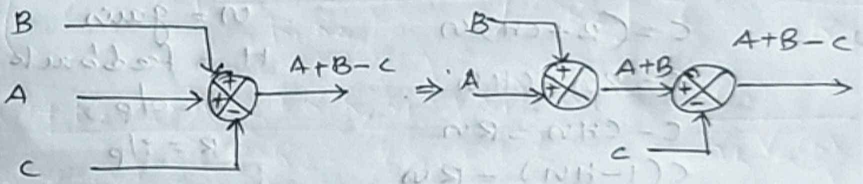
Rule-6: moving the summing point before the block.



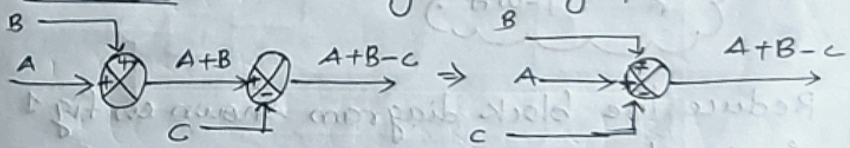
Rule-7: Interchanging the summing point.



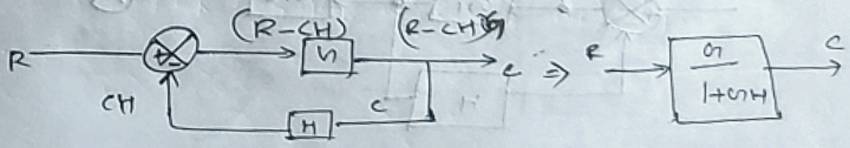
Rule-8: Splitting summing point.



Rule-9: combining summing point.



Rule-10: Elimination of (negative) feedback loop



$C = (R - CH)G$

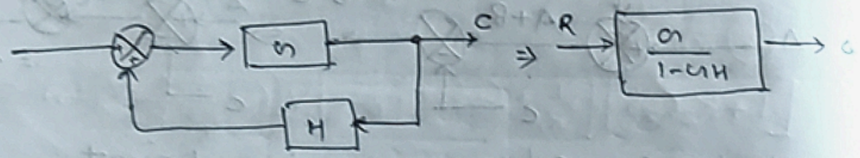
$C = RG - CHG$

$C + CHG = RG$

$C(1 + HG) = RG$

$$\frac{C}{R} = \frac{G}{1+GH}$$

Rule-11: elemenation of (+ve) feedback loop



$C = (R + CH)G.$

$C = (R + CH)G$

$C = RG + CHG$

$C - CHG = RG$

$C(1 - HG) = RG.$

$$\frac{C}{R} = \frac{G}{1-GH}.$$

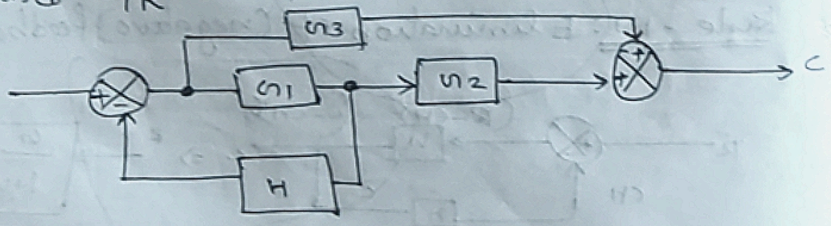
G = gain

H = feedback gain

C = o/p

R = i/p

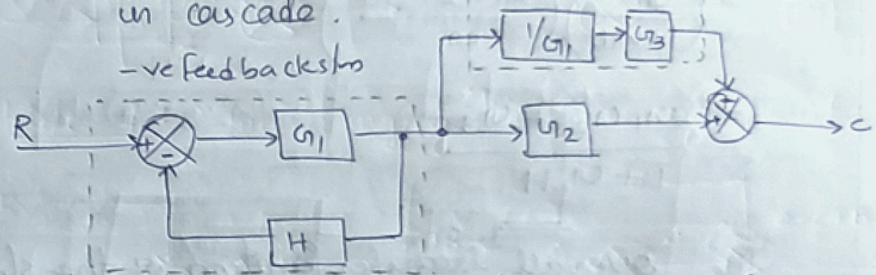
Q) Reduce the block diagram shown in fig 1 and find C/R



step- 1

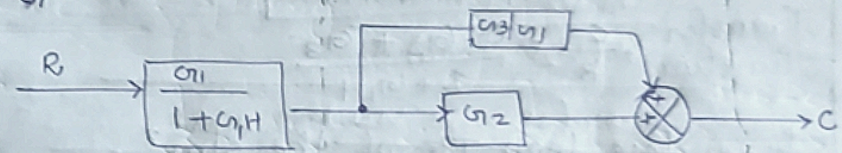
move the branch point after the block situated to the G_1 ~~to~~ left side. to the right side

Step 2: Eliminate the feedback path & combining blocks in cascade.



Here $1/G_1$ and G_3 are in series $= \frac{G_3}{G_1}.$

$\frac{G_3}{G}$ step 3 : combining parallel blocks

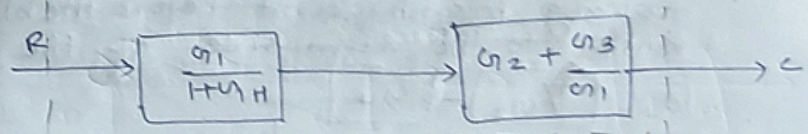


Here the block G_3/G_1 and G_2 are parallel.

then the result is $\frac{G_3}{G_1} + G_2.$

In the starting part of the block diagram contain G_1 and h blocks. The total portion is -ve feed back s/m. & result is $\frac{G_1}{1+G_1H}.$

step 4: combining blocks in cascade.



Here we have 2 blocks and they are series

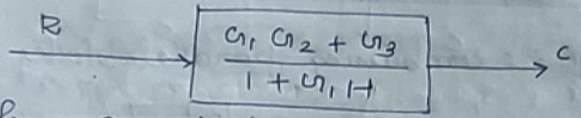
Transfer function $= \frac{C}{R}$

$$\frac{C}{R} = \left(\frac{G_1}{1+G_1H}\right)\left(G_2 + \frac{G_3}{G_1}\right)$$

$$= \left(\frac{\cancel{G_1}}{1+G_1H}\right)\left(\frac{G_1G_2 + G_3}{\cancel{G_1}}\right)$$

$$= \underline{\underline{\frac{G_1G_2 + G_3}{1+G_1H}}}$$

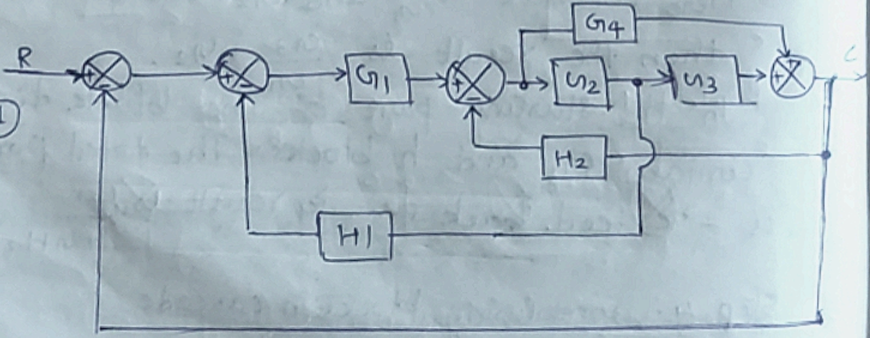
The overall transfer function of the s/m,



Transfer function, $\frac{C}{R} = \frac{G_1G_2 + G_3}{1+G_1H}$

Q) find close loop TF of the s/m?

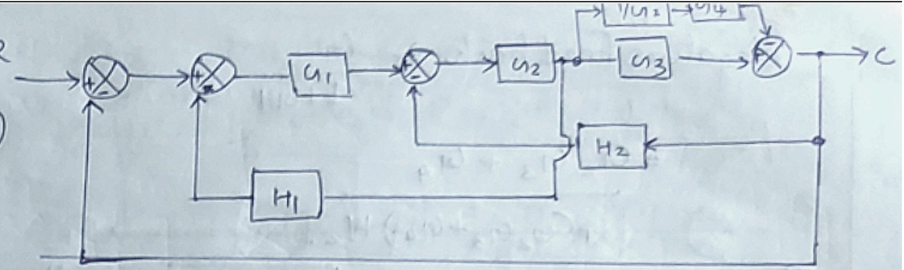
①



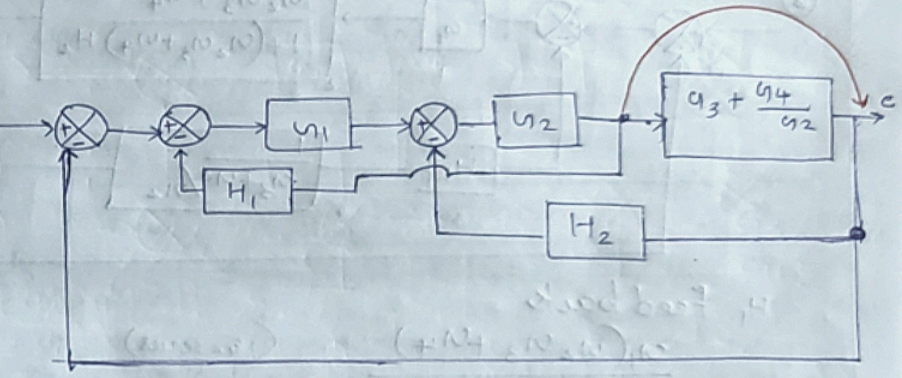
moving branch point before the block and transfer to the right side of the point

ie, $\boxed{1/G_2} \rightarrow \boxed{G_4}$.

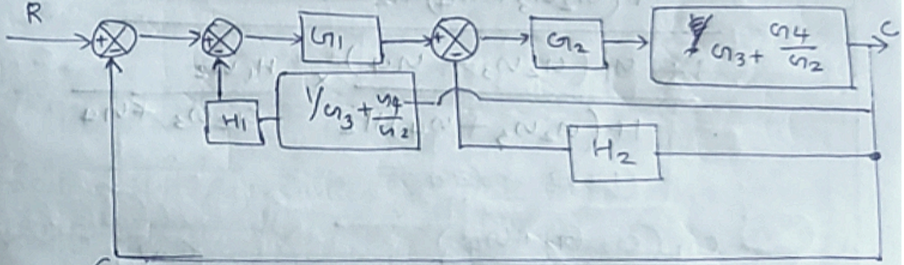
②



③

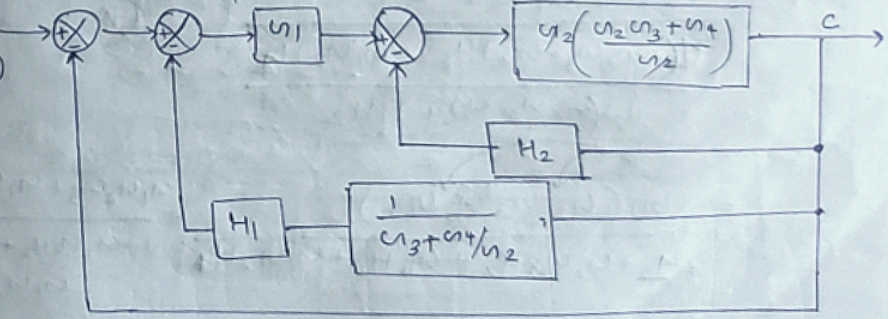


④



(G_2 & G_3/G_4 series and take LCM)

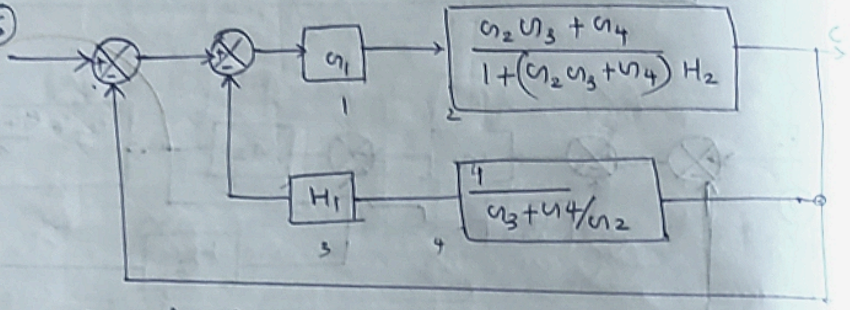
⑤



Negative Feed back ⇒ $\frac{G}{1+GH}$

$$= \frac{G_2G_3+G_4}{1+(G_2G_3+G_4)H_2}.$$

⑥



H_1 Feed back

(1&2 series)

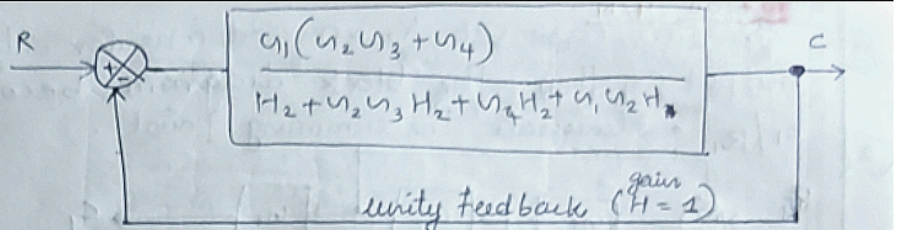
$$= \frac{\frac{G_1(G_2G_3+G_4)}{1+(G_2G_3+G_4)H_2}}{1+\frac{G_1(\cancel{G_2G_3+G_4})}{1+(G_2G_3+G_4)H_2}\times\frac{H_1G_2}{\cancel{G_2G_3+G_4}}}$$

(1+2) × (3+4)

$$= \frac{\frac{G_1(G_2G_3+G_4)}{1+\cancel{(G_2G_3+G_4)H_2}}}{\frac{1(+G_2G_3+G_4)H_2+G_1G_2H_1}{1+\cancel{(G_2G_3+G_4)}H_2}}$$

$$\frac{x/y}{p/q} = x/y \times q/p$$

$$= \frac{G_1(G_2G_3+G_4)}{H_2+G_2G_3H_2+G_4H_2+G_1G_2H_1} = \frac{G_1G_2G_3+G_1G_4}{1+G_2G_3H_2+G_4H_2+G_1G_2H_1}$$



Here the Feedback have no gain blocks It Indicate unity feedback s/m. ie, H = 1

$$\frac{\frac{G_1G_2G_3+G_1G_4}{H_2+G_2G_3H_2+G_4H_2+G_1G_2H_1}}{1+\frac{G_1G_2G_3+G_1G_4}{1+G_2G_3H_2+G_4H_2+G_1G_2H_1}\times 1}$$

$$\frac{x/y}{p/q} = x/y \times q/p$$

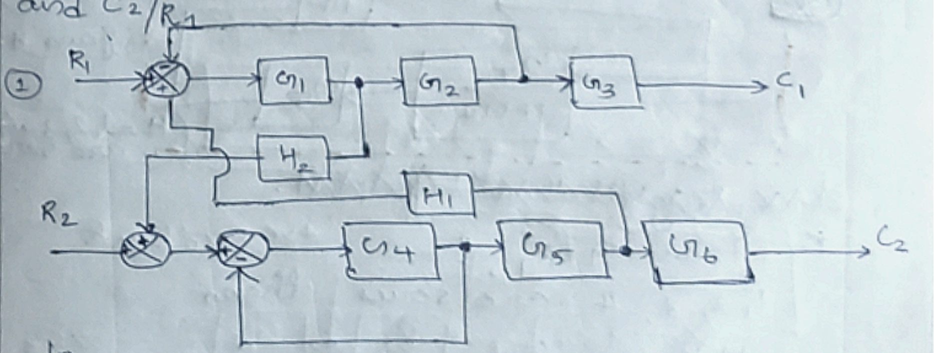
(LCM) edukka.

$$\frac{C}{R} = \frac{G_1G_2G_3+G_1G_4}{(1+G_1G_3H_2+G_4H_2+G_1G_2H_1+G_1G_2G_3+G_1G_4)}$$

Transfer function = $\frac{C}{R}$

Q) For the s/m represented by the block diagram, find C_1/R_1 and C_2/R_1

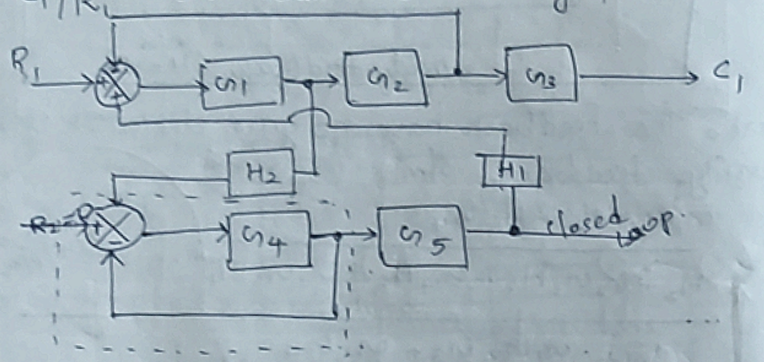
①



$\underline{\underline{Sol^n}}$: we have 2 input ie, R_1 and R_2 and 2 output C_1 and C_2.

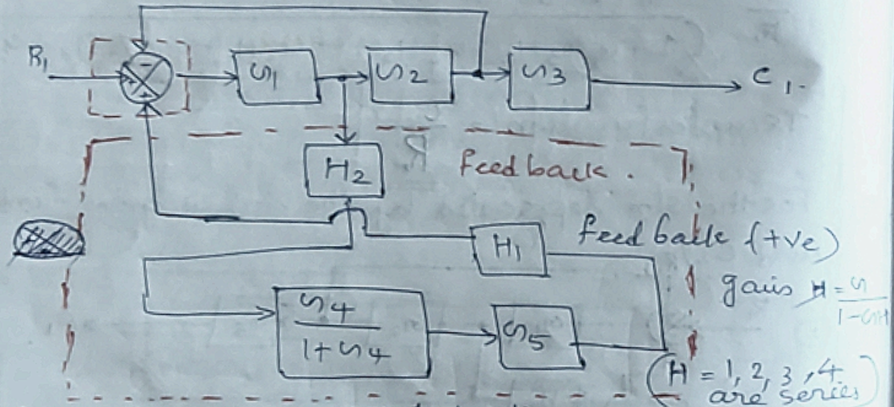
To find G_1/R_1

In this case set $R_2 = 0$. and consider only one output C_1 then the block diagram becomes C_1/R_1. • eliminate the summing point.



combining the blocks in cascade and splitting the summing point

$G_4 \rightarrow \frac{G_4}{1+G_4}$.



~~[illegible].~~

to find G_1/R_1 ?

1, 2, 3, 4 are series then,

$$= H_1 H_2 G_3 \times \frac{G_4}{1+G_4}$$

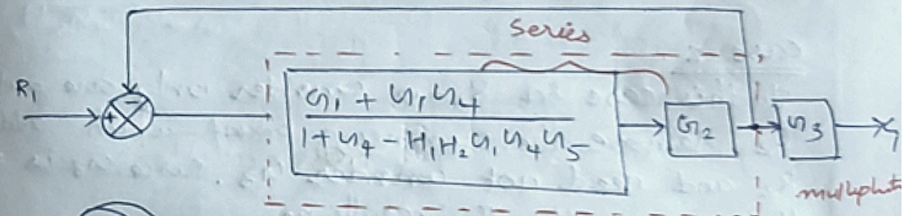
$$= \frac{H_1 H_2 G_5 G_4}{1+G_4}$$ consider the +ve feedback

$$= \frac{G_1}{1 - \frac{G_1 H_1 H_2 G_4 G_5}{1+G_4}}$$

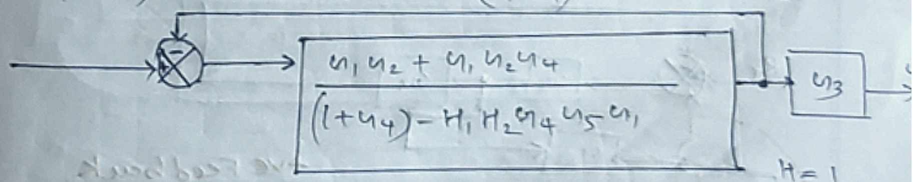
$$= \frac{G_1}{\frac{(1+G_4) - (G_2 H_1 H_2 G_4 G_5)}{1+G_4}} \quad \begin{matrix} x \\ p \\ q \end{matrix}$$

$$\frac{x}{(p/q)} = x \times q/p.$$

$$= \frac{G_1 (1+G_4)}{1+G_4 - H_1 H_2 G_1 G_4 G_5} = \frac{G_1 + G_1 G_4}{1+G_4 - H_1 H_2 G_1 G_4 G_5}$$



$$\frac{G_1 G_2 (1+G_4)}{(1+G_4) - H_1 H_2 G_1 G_4 G_5} = \frac{G_1 G_2 + G_1 G_2 G_4}{(1+G_4) - H_1 H_2 G_1 G_4 G_5}$$



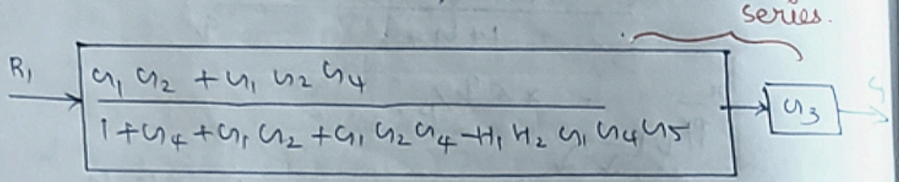
$H = 1$

-ve Feedback $\frac{G}{1+GH}$

$$R_1 \rightarrow \frac{\frac{G_2 G_1 + G_2 G_4 G_1}{(1+G_4) - H_1 H_2 G_1 G_4 G_5}}{1 + \frac{G_1 G_2 + G_1 G_2 G_4}{1+G_4 - H_1 H_2 G_1 G_4 G_5} \times 1}$$

$$\frac{\dfrac{G_1G_2+G_1G_2G_4}{1+G_4-H_1H_2G_1\cancel{G_4}G_5}}{\dfrac{1+G_4-H_1H_2G_1G_4G_5+G_1G_2+G_1G_2G_4}{1+G_4-H_1H_2\cancel{G_4}G_4G_5}}$$

$$\frac{C_1}{R_1} = \frac{G_1G_2+G_1G_2G_4}{1+G_4+G_1G_2+G_1G_2G_4-H_1H_2G_1G_4G_5}$$

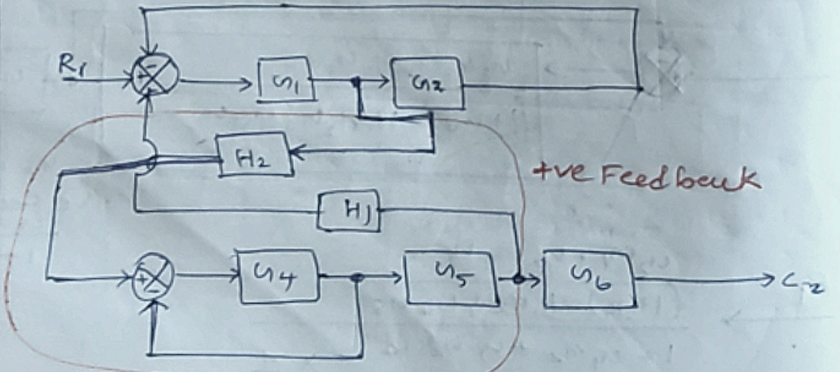


$$\frac{C_1}{R_1} = \frac{G_1G_2G_3+G_1G_2G_3G_4}{1+G_4+G_1G_2+G_1G_2G_4-H_1H_2G_1G_4G_5}$$

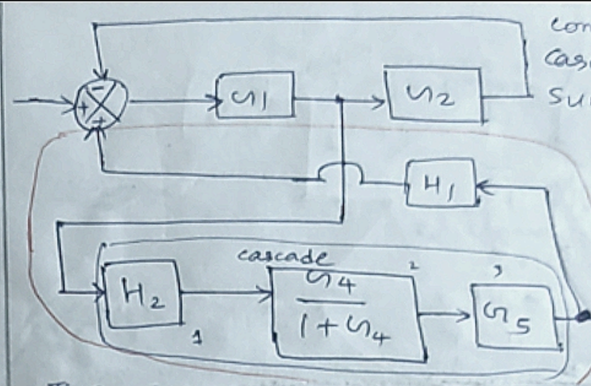
<u>case-2</u>.

In this case set $R_2=0$ consider only one output C_2: remove the summing point which adds R_2 and need not consider G_3. since G_3 is on the open path. The resulting

<u>step-1</u>: Elimenale the feedback path.



②



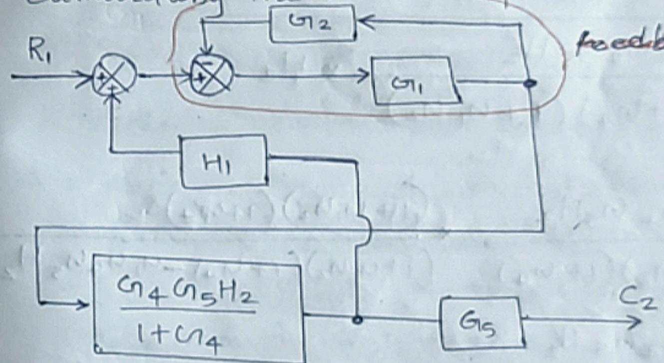
combining blocks in cascade & splitting the summing point.

+ve Feedback.

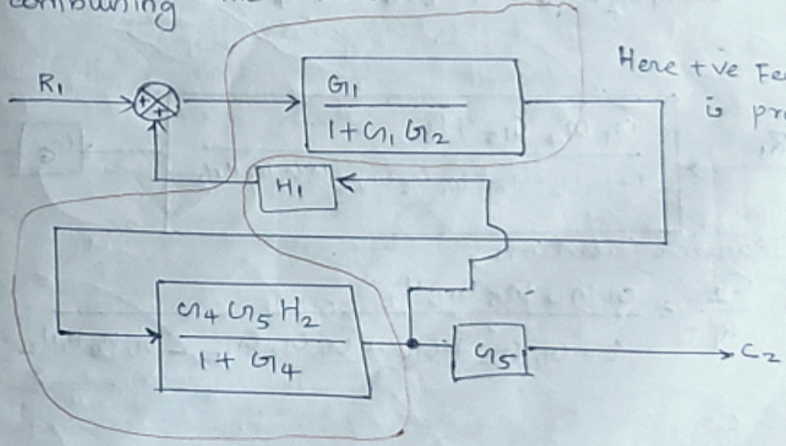
(1, 2, & 3 are series)

Th.

③ Eliminating the feedback path.

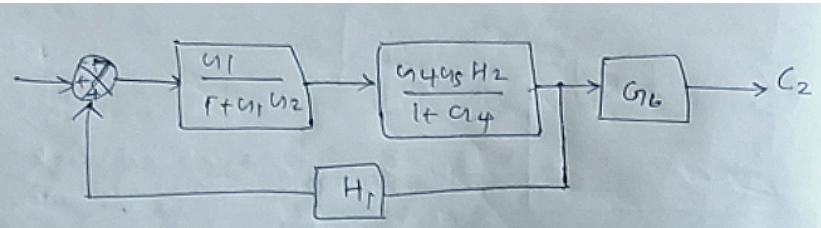


④ combining the blocks in cascade.



Here +ve Feedback is present

④

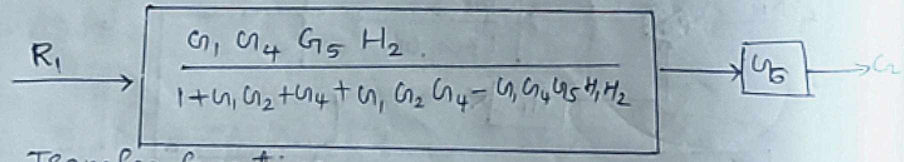


+ve feedback ⇒ $\frac{G}{1 - GH}$

$$= \frac{\dfrac{G_1G_4G_5H_2}{(1+G_1G_2)(1+G_4)}}{1 - \dfrac{G_1G_4G_5H_2}{(1+G_4)(1+G_1G_2)} \times H_1}$$

$$= \frac{G_1G_4G_5H_2}{(1+G_1G_2)(1+G_4)} \times \frac{(1+G_1G_2)(1+G_4)}{(1+G_1G_2)(1+G_4) - G_1G_4G_5H_2}$$

$$= \frac{G_1G_4G_5H_2}{1+G_1G_2+G_4+G_1G_2G_4 - G_1G_4G_5H_1H_2}$$



Transfer function,

$$\frac{C_2}{R_1} = \frac{G_1G_4G_5G_6H_2}{1+G_1G_2+G_4+G_1G_2G_4 - G_1G_4H_1H_2}$$

Signal flow graph.

It is used to represent the control s/m graphically

imp sure Q.. MASON'S GAIN FORMULA.

It is used to determine the Transfer function of the s/m from the signal flow graph. consider, R(s) is the i/p to the s/m and C(s) is the o/p of the s/m. then, the Transfer function

$$T = \frac{1}{\Delta} \sum_K P_K \Delta_K$$

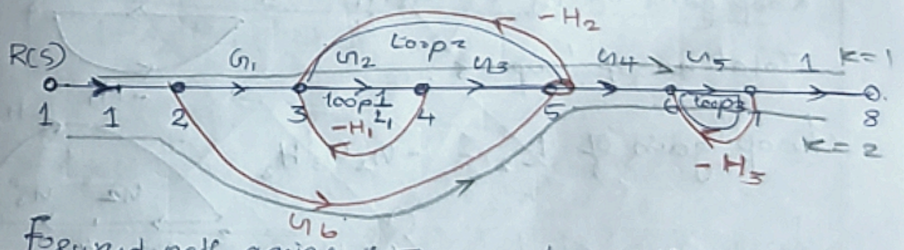
where, P_K = Forward path gain of K^{th} forward path

K = No. of forward path in the signal flow grap

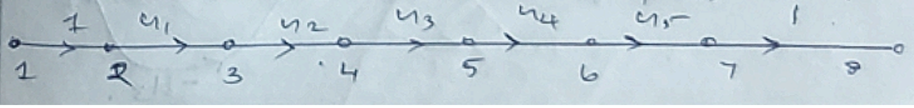
Δ = 1 − (sum of individual loop gains) + (sum of gain product of all possible combination of 2 non-touching loop) − (sum of gain products of all possible combination of three non-touching loops) + ...

Δ_K = Δ for that part of the graph which is not touching K^{th} forward path.

Q, Find the overall transfer function of the s/m whose signal flow graph is



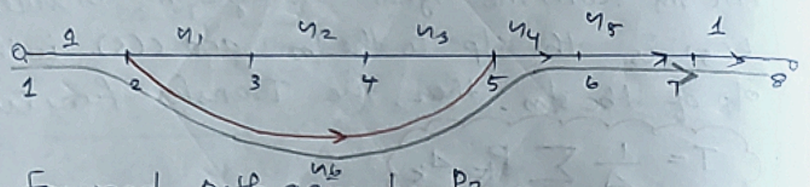
Forward path gains : There are two paths K = 2.



let forward path gain be P_1

$P_1 = 1 \times G_1 \times G_2 \times G_3 \times G_4 \times G_5 \times 1$.

$P_1 = G_1 G_2 G_3 G_4 G_5$



forward path gain be P_2.

$P_2 = 1 \times G_6 \times G_4 \times G_5 \times 1$

$P_2 = G_6 G_5 G_4$.

Transfer function $(T) = \frac{1}{\Delta} \sum_K P_K \Delta_K$

$\sum_K P_K \Delta_K = \sum_{K=2} P_K \Delta_K$.

$= P_1 \Delta_1 + P_2 \Delta_2$

$\Delta = 1 -$ sum of ind. loop gain + (sum of gain produced of a non-touching loop) − (sum of gain produced by 3 non-touching loop)

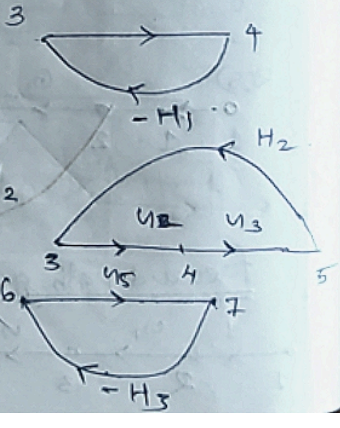
Loops.

Loop gain of $L_1 = -G_2 H_1$

Total no. of loop = 3.

Loop gain of $L_2 = -G_2 G_3 H_2$

Loop gain of $L_3 = -G_5 H_3$



Sum of Loop gains = $-G_2H_1 - G_2G_3H_2 - G_5H_3$

non-touching loops.

L_1 and L_2 are touching loop.

L_1 and L_3 are non-touching loop.

L_2 and L_3 are non-touching Loop.

Here we have 2 set of non touching loop.

then, $\Delta = 1 - (-G_2H_1 - G_2G_3H_2 - G_5H_3) + \{(-G_2H_1 \times -G_5H_3) + (-G_2G_3H_2 \times -G_5H_3)\}$

$\Delta = 1 + (G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_5G_3H_2H_3$

$P_1 = G_1G_2G_3G_4G_5$

$P_2 = G_4G_5G_6$

Δ_1 = non-touching loop in 1st Forward path.

1 − (non touching Loop gain) = 1 − 0 = 1

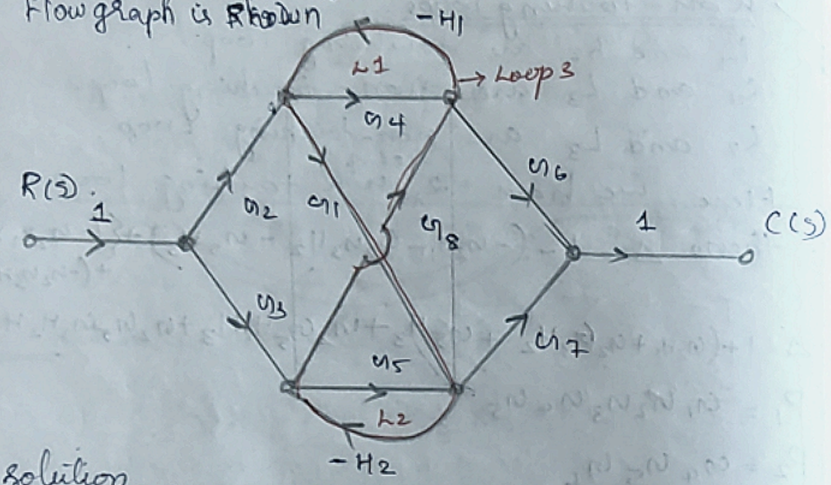
Δ_2 = Non-touching loop in 2nd forward path.

$= 1 - (-G_2H_1) = 1 + G_2H_1$

$T.F = \frac{1}{\Delta}\{\Delta_1 P_1 + \Delta_2 P_2\}$

$$= \frac{(G_1G_2G_3G_4G_5)1 + (G_4G_5G_6)(1+G_2H_1)}{1+G_2H_1 + G_2G_3H_2 + G_5H_3 + G_3H_1G_5H_3 + G_2G_3G_5H_2H_3}.$$

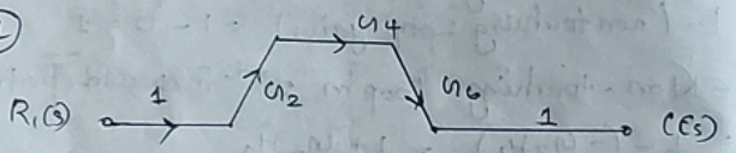
Q) Find the overall gain of the system. The signal flowgraph is ~~R~~shown



Solution

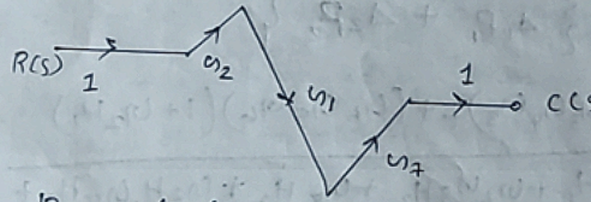
No. of forward path = 6 :

①



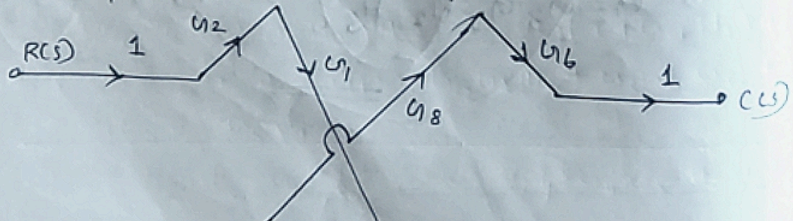
$P_1 = G_2 G_4 G_6$

②



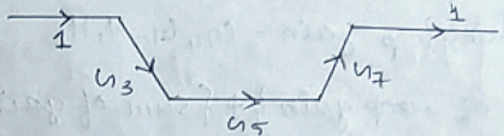
$P_2 = G_1 G_2 G_7$

③



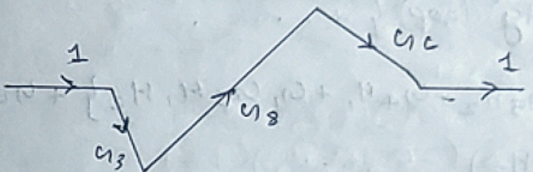
$P_3 = -G_1 G_2 G_8 H_2$

④



$P_4 = G_3 G_5 G_7$

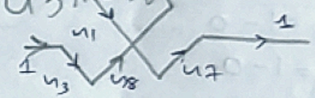
⑤



$P_5 = G_3 G_8 G_6$

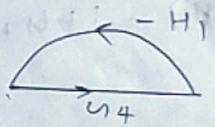
⑥

$P_6 = -G_1 G_3 G_7 G_8 H_1$ → $G_3 \times G_8 \times -H_1 \times G_1 \times G$

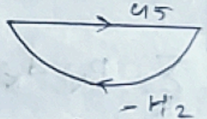


No. of Loop = 3

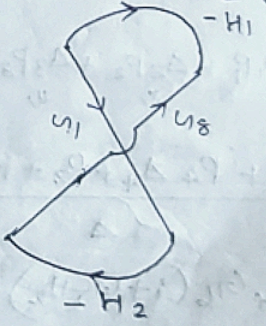
$L_1 = -G_4 H_1$



$L_2 = -G_5 H_2$.



$L_3 = ^{+} G_1 H_2 G_8 L_1$



Non-touching loop:

L_1 and L_2 are non-touching loop.

L_1 and L_3 are touching loop

L_2 and L_3 are touching loop

one non-touching loop gain = $G_4 G_5 H_1 H_2$

$\Delta = 1 - \{$sum of loop gain$\} + \{$sum of gain prod of 2 non-touching loop$\} - \{$sum of gain product of 3 non-touching loop$\} + \dots$

$\Delta = 1 - \{-G_5 H_2 - G_4 H_1 + G_1 G_8 H_1 H_2\} + G_4 G_5 H_1 H_2$

$\Delta_1 = 1 - (-G_5 H_2)$

$= 1 + G_5 H_2$.

$\Delta_2 = 1 - 0$.

$\Delta_3 = 1 - 0$

$\Delta_4 = 1 - (-G_4 H_1) = 1 + G_4 H_1$.

$\Delta_5 = 1$

$\Delta_6 = 1$

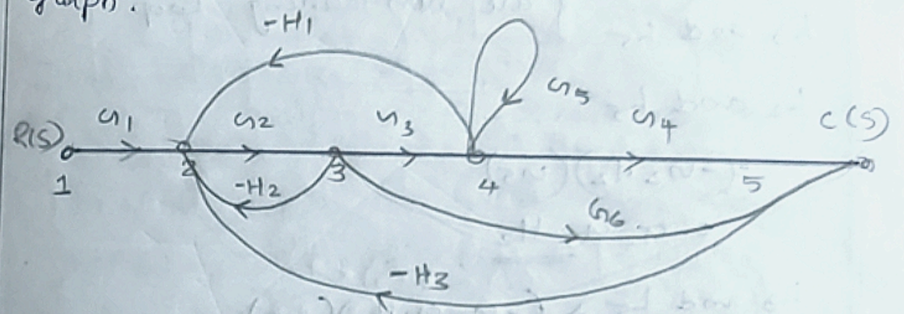
$$T.F = \frac{1}{\Delta} \sum_{k} P_K \Delta_K$$

$$= \frac{1}{\Delta} \{ \Delta_1 P_1 + \underset{\Downarrow}{\Delta_2 P_2} + \underset{\Downarrow}{\Delta_3 P_3} + \underset{\Downarrow}{\Delta_4 P_4} + \underset{\Downarrow}{\Delta_5 P_5} + \underset{\Downarrow}{\Delta_6 P_c} \}$$

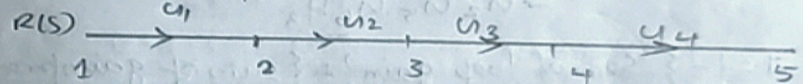
$$TF = \frac{P_1 \Delta_1 + P_4 \Delta_4 + P_2 + P_3 + P_5 + P_L}{\Delta}$$

$$TF = \frac{G_2 G_4 G_6 (1 + G_5 H_2) + G_3 G_5 G_7 (1 + G_4 H_1) + - G_1 G_2 G_8 H_2 + - G_1 G_2 G_6 G_8 H_2 + G_3 G_8 G_6 + - G_1 G_3 G_7 G_8 H_1}{1 - \{-G_5 H_2 - G_4 H_1 + G_1 G_8 H_1 H_2\} + G_4 G_5 H_1 H_2}$$

Q, find the over all gain $C(s)/R(s)$ for the signal flow graph.

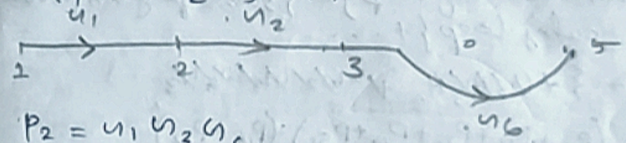


1. forward path = 2.



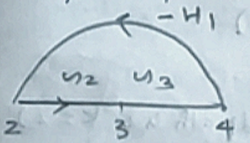
$P_1 = G_1 G_2 G_3 G_4$.

2)



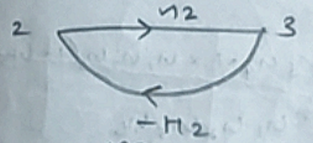
$P_2 = G_1 G_2 G_c$

No. of Loop = 5



loop gain · $R_1 = -G_2 G_3 H_1$

P_{11}

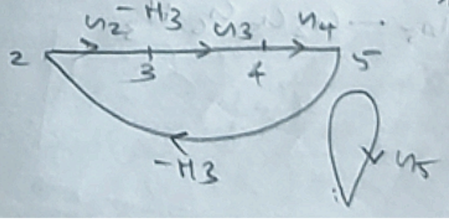


$R_2 = -H_2 G_2$.

P_{21}

$R_3 = -G_2 G_6 H_3$

P_{31}



$R_4 = -G_2 G_3 G_4 H_3$

P_{41}

$R_5 = G_5$

P_{51}

L_2 and L_5
L_3 and L_5 } are non-touching Loop

L_2 and L_5

$= (-G_2 H_2)(G_5)$

$= -G_2 G_5 H_2$

L_3 and $L_5 \Rightarrow (-G_2 G_6 H_3)(G_5)$

$= G_2 G_5 G_6 H_3$

$\Delta = 1 - \{$ Sum of loop gain $\} + \{$ sum of gain product of 2 non-touching loop $\} - \{$ sum of gain product of 3 non touching loop $\} + \cdots$

$\Delta = 1 - \{ \cancel{+G_2 G_5 H_2} \; \cancel{+ - G_2 G_5 G_6 H_3}$

$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41} + P_{51}) + (P_{12} + P_{22})$

$= 1 - (-G_2 G_3 H_1 - H_2 G_2 - G_2 G_3 G_4 H_3 + G_5 - G_2 G_6 H_3) +$
$(-G_2 H_2 G_5) - G_2 G_5 G_6 H_3)$

$\Delta_2 = 1 - G_5$

$T = \frac{1}{\Delta} \sum_{K} P_K \Delta_K$ (no. of forward path is 2 K = 2)

$\frac{1}{\Delta}[P_1 \Delta_1 + P_2 \Delta_2] = \frac{1}{\Delta}[G_1 G_2 G_3 G_4 \times 1 \times G_1 G_2 G_6 (1 - G_5)]$

$$= \frac{G_1 G_2 G_3 G_4 + G_1 G_2 G_6 - G_1 G_2 G_5 G_6}{1 + G_2 G_3 H_1 + H_2 G_2 + G_2 G_3 G_4 H_3 - G_5 + G_2 G_6 H_3 - G_2 H_2 G_5 - G_2 G_5 G_6 H_3}$$

CHAPTER-2

TIME RESPONSE ANALYSIS

- **Time response Analysis.**

The Time response of a s/m is the o/p of the closed loop s/m as a function of time. Here $TF = C(S)/R(S)$. from here $C(S) = R(S) \cdot TF$

$c(t) =$ ILT of $C(S)$

$= L^{-1}\{ C(S) \}$

$= L^{-1}\{ R(S)\ TF \}$

- **Order of a system.**

consider the i/p & o/p of relationship a control s/m. In DE form For eg:-

$$a_0 \frac{d^n}{dt^n} p(t) + a_1 \frac{d^{n-1}}{dt^{n-1}} p(t) + \cdots\cdots + a_{n-1} \frac{d}{dt} p(t) + a_n p(t) = b_0 \frac{d^m}{dt^m} q(t)$$
$$+ b_1 \frac{d^{m-1}}{dt^{m-1}} q(t) + b_2 \frac{d^{m-2}}{dt^{m-2}} q(t) + \cdots\cdots + b_{m-1} \frac{d}{dt} q(t) + b_m q(t)$$

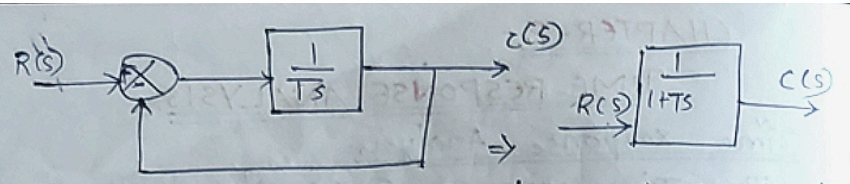
where $p(t) =$ output / Response

$q(t) =$ input / Excitation.

The order of the s/m is given by the order of DE. Here, the s/m is n^{th} order DE. then the s/m is called n^{th} order DE, then the s/m is called n^{th} order system.

- **Response of I^{st} order s/m for unit step input.**

The closed loop First order s/m with unity feedback -ve feedback $\Rightarrow \frac{G}{1+GH} = \frac{1/Ts}{1 + 1/Ts \times 1} \Rightarrow \frac{1}{Ts+1}$



The closed loop transfer function of first order s/m

$$\frac{C(S)}{R(S)} = \frac{1}{1+TS}.$$

The out put $C(S) = R(S) \times \frac{1}{1+TS}$.

Here R(S) is unit step input. $R(S) = \frac{1}{S}$

The response in s-domain, $C(S) = R(S)\frac{1}{1+TS}$.

$$= \frac{1}{S}\frac{1}{(1+TS)} = \frac{1}{ST\left(\frac{1}{T}+S\right)} = \frac{\frac{1}{T}}{S\left(S+\frac{1}{T}\right)}$$

By partial fraction expansion,

$$C(S) = \frac{\frac{1}{T}}{S\left(S+\frac{1}{T}\right)} = \frac{A}{S} + \frac{B}{\left(S+\frac{1}{T}\right)}$$

A is obtained by multiply (CS) by s and letting s = 0.

$$A = C(S) \times S\Big|_{S=0} \; \frac{\frac{1}{T}}{S\left(S+\frac{1}{T}\right)} \times S\Bigg|_{S=0} = \frac{\frac{1}{T}}{S+\frac{1}{T}}\Bigg|_{S=0} = \frac{\frac{1}{T}}{\frac{1}{T}} = 1$$

B is obtained by multiply C(S) by $(S+1/T)$ and letting $s=1$

$s = -1/T$

$$B = C(S) \times \left(S+\frac{1}{T}\right)\Big|_{S=\frac{1}{T}} = \frac{\frac{1}{T}}{S\left(S+\frac{1}{T}\right)} \times \left(S+\frac{1}{T}\right)\Bigg|_{S=\frac{1}{T}} = \frac{\frac{1}{T}}{S}\Bigg|_{S=1/T}$$

$$1/T \big/ -1/T = -1$$

$$C(S) = \frac{1}{S} - \frac{1}{S+\frac{1}{T}}$$

The response in time domain is given by

$$c(t) = L^{-1}\{C(S)\}$$

$$= L^{-1}\left\{\frac{1}{S} - \frac{1}{S+\frac{1}{T}}\right\} = 1 - e^{t/T}$$

for closed loop I st order s/m, unit step response $= 1 - e^{t/T}$

step response $= A\left(1 - e^{-t/T}\right)$

→ **PF. by Another method**

$$c(t) = L^{-1}\left\{\frac{1}{S(1+TS)}\right\}$$

Apply PF in $\frac{1}{S(1+TS)}$

$$\frac{1}{S(1+TS)} = \frac{A}{S} + \frac{B}{1+ST} \longrightarrow ⓐ$$

⊗ a with $s(1+ST)$

$1 = A(1+ST) + BS$

$1 = A + AST + BS$

$1 = A + (AT+B)S$ equalting coeffient of const-terms $\boxed{1 = A.}$

Equalting the coeffs of s.

$0 = AT + B$

$AT = -B$

$\boxed{B = -AT = -T}$

$$\frac{1}{S(1+ST)} = \frac{1}{S} + \frac{-T}{1+ST}$$

$$① \Rightarrow c(t) = L^{-1}\left\{\frac{1}{s} - \frac{T}{1+sT}\right\}$$

$$c(t) = L^{-1}\left\{\frac{1}{s}\right\} - L^{-1}\left\{\frac{T}{1+sT}\right\}$$

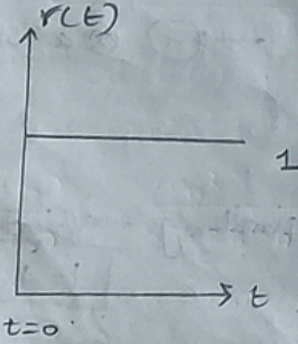
$$c(t) = 1 - L^{-1}\left\{\frac{\cancel{T}}{\cancel{T}(1/T + s)}\right\}$$

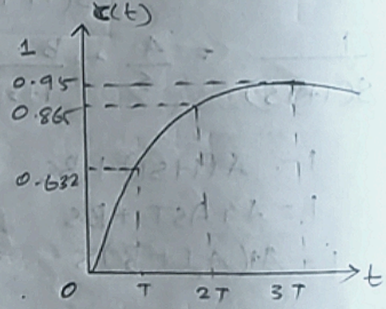
$$c(t) = 1 - L^{-1}\left\{\frac{1}{s+\frac{1}{T}}\right\}$$

$$\therefore c(t) = \underline{\underline{1 - e^{-t/T}}}$$

when $t=0$, $c(t) = 1 - e^{0} = 0$.

$t = 1$, $c(t) = 1 - e^{-1} = 0.632$





Response of I st order s/m for unit ramp input.

The closed loop Transfer function of first order s/m,

$$\boxed{\frac{1}{s} \xrightarrow{\text{step}} t}$$

$$\frac{C(S)}{R(S)} = \frac{1}{1+Ts}. \qquad c(t) = L^{-1}\{C(S)\} \qquad L[t^n] = \frac{n!}{s^{n+1}}$$

The output $C(S) = R(S) \times \frac{1}{1+Ts}$.

Here $R(S)$ is unit ~~step~~ ramp input $R(S) = 1/s^2$

The response in s-domain $C(s) = R(S)\frac{1}{1+Ts}$.

$$= \frac{1}{s^2} \cdot \frac{1}{1+Ts} = \frac{1}{s^2(1+Ts)}$$

Let, Taking partial fraction method.

$$\frac{1}{s^2(1+Ts)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(1+Ts)} \longrightarrow ⓐ$$

$$A = C(S) \times s \Big|_{s=0} = \frac{1}{s^2(1+Ts)} \times s = -T$$

$$B = C(S) \times s^2 \Big|_{s^2} = \frac{1}{s^2(1+Ts)} \times s^2 \quad \frac{1}{s^2+Ts^3} \times s^2 = -1$$

$$C = C(S) \times (1+Ts) \Big|_{s \Rightarrow} \frac{1}{s^2(1+Ts)} \times (1+Ts) = \underline{\underline{T^2}}$$

$$\frac{1}{s^2+Ts^3}(1+Ts)$$

$$s^2 + Ts^2.$$

ⓐ becomes,

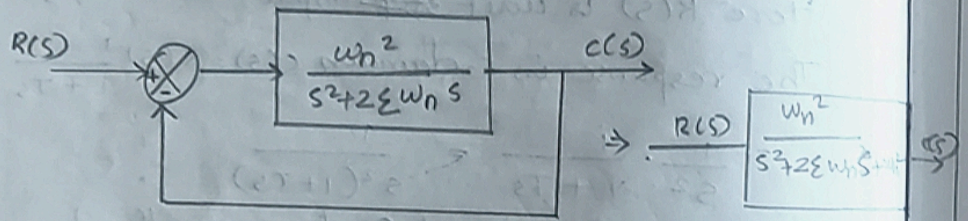
$$c(t) = L^{-1}\left\{\frac{-T}{s} + \frac{-1}{s^2} + \frac{T^2}{1+Ts}\right\}$$

$$c(t) = L^{-1}\left\{\frac{-T}{s}\right\} - L^{-1}\left\{\frac{1}{s^2}\right\} + L^{-1}\left\{\frac{T^2}{1+Ts}\right\}$$

$$c(t) = -T + t + Te^{(-t/T)}$$

$$\underline{\underline{c(t) = t + Te^{(-t/T-1)}}}$$

Second order system. (ε - Setta)



The standard form of closed loop transfer function of second order system is given by,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\varepsilon\omega_n s+\omega_n^2}.$$

where, ω_n = undamped natural frequency rad/sec.

actual damping
critical damping

⇐ (ξ) ε = Damping ratio

The damping ratio is defined as the ratio of actual damping to the critical damping. The response c(t) of the s/m depends on the value of damping ratio. based on this s/m can be classified as,

conditions:

1) undamped s/m $\varepsilon = 0$.
2) under damped s/m $0 < \varepsilon < 1$ (·1 – ·9)
3) critically damped s/m $\varepsilon = 1$
4) over damped s/m $\varepsilon > 1$

Here, the denomenator term of T.F is,

$\cdot s^2 + 2\varepsilon\omega_n s + \omega_n^2 = 0$. This is known as characterstic eqn.

The roots of characterstic eqns are,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = s^2 \cdot (\text{coeff } s^2) = 1$
$b = 2\varepsilon\omega_n$ (coeff of s)
$c = \omega_n^2$.

$$x = \frac{-2\varepsilon\omega_n \pm \sqrt{2\varepsilon\omega_n^2 - 4\omega_n^2}}{2\times1}$$

$$x = \frac{-\cancel{2}\varepsilon\omega_n \pm \sqrt{\cancel{4}\omega_n^2(\varepsilon^2-1)}}{\cancel{2}}$$

$\boxed{i = \sqrt{-1}}$

$$\underline{\underline{x = -\varepsilon\omega_n \pm \omega_n\sqrt{\varepsilon^2-1}}}.$$

when $\varepsilon = 0$, $s_1, s_2 = \pm i\omega_n$; {roots are purely imaginary & the s/m is undamped.

when $\varepsilon = 1$, $s_1, s_2 = -\omega_n$; {roots are real & equal & the s/m is critically damped.

when $\varepsilon > 1$, $s_1, s_2 = -\varepsilon\omega_n \pm \omega_n\sqrt{\varepsilon^2-1}$
roots are real and unequal & the s/m is over damped

when $0 < \varepsilon < 1$, $s_1, s_2 = -\varepsilon\omega_n \pm \omega_n\sqrt{\varepsilon^2-1}$

ⓐ becomes,

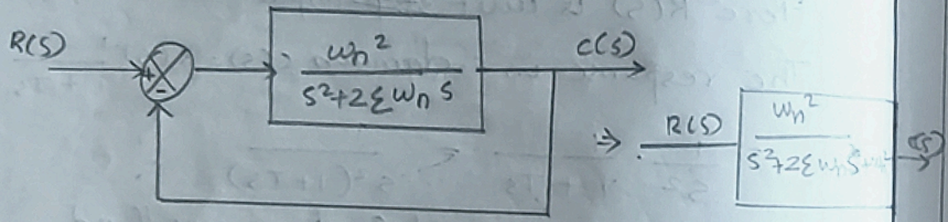
$$c(t) = L^{-1}\left\{\frac{-T}{s} + \frac{-1}{s^2} + \frac{T^2}{1+Ts}\right\}$$

$$c(t) = L^{-1}\left\{\frac{-T}{s}\right\} - L^{-1}\left\{\frac{1}{s^2}\right\} + L^{-1}\left\{\frac{T^2}{1+Ts}\right\}$$

$$c(t) = -T + t + Te^{(-t/T)}$$

$$\underline{\underline{c(t) = t + Te^{(-t/T-1)}}}$$

Second order system. (ε - Setta)



The standard form of closed loop transfer function of second order system is given by,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\varepsilon\omega_n s+\omega_n^2}.$$

critical damping.
critical damping

where, ω_n = undamped natural frequency rad/sec.

⇐ (ξ) ε = Damping ratio

The damping ratio is defined as the ratio of actual damping to the critical damping.

The response c(t) of the s/m depends on the value of damping ratio. based on this s/m can be classified as,

conditions:

1) undamped s/m $\varepsilon = 0$.
2) under damped s/m $0 < \varepsilon < 1$ (·1 - ·9)
3) critically damped s/m $\varepsilon = 1$
4) over damped s/m $\varepsilon > 1$

Here, the denomenator term of T.F is,

$s^2 + 2\varepsilon\omega_n s + \omega_n^2 = 0$. This is known as characteristic eqn.

The roots of characteristic eqns are,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \left\{\begin{array}{l} a = s^2 \cdot (\text{coeff } s^2) = 1 \\ b = 2\varepsilon\omega_n \text{ (coeff of } s) \\ c = \omega_n^2. \end{array}\right.$$

$$x = -\frac{2\varepsilon\omega_n \pm \sqrt{2\varepsilon\omega_n^2 - 4\cancel{4}^{1}\omega_n^2}}{2\times1}$$

$i = \sqrt{-1}$

$$x = -\frac{\cancel{2}\varepsilon\omega_n \pm \sqrt{\cancel{4}\omega_n^2(\varepsilon^2-1)}}{\cancel{2}}$$

$$\underline{\underline{x = -\varepsilon\omega_n \pm \omega_n\sqrt{\varepsilon^2-1}}}.$$

when $\varepsilon = 0$, s_1, $s_2 = \pm i\omega_n$; {roots are purely imaginary & the s/m is undamped.

when $\varepsilon = 1$, s_1, $s_2 = -\omega_n$; {roots are real & equal & the s/m is critically damped.

when $\varepsilon > 1$, $s_1, s_2 = -\varepsilon\omega_n \pm \omega_n\sqrt{\varepsilon^2-1}$

roots are real and unequal & the s/m is over damped

when $0 < \varepsilon < 1$, s_1, $s_2 = -\varepsilon\omega_n \pm \omega_n\sqrt{\varepsilon^2-1}$

$$= -\varepsilon\omega_n \pm \omega_n\sqrt{(-1)(1-\varepsilon^2)}$$

$$= -\varepsilon\omega_n \pm \omega_n\sqrt{-1}\,\sqrt{1-\varepsilon^2} = -\varepsilon\omega_n \pm j\omega_n\sqrt{1-\varepsilon^2}$$

$$= -\varepsilon\omega_n \pm j\omega_d; \left\{\begin{array}{l}\text{root are complex conjugate}\\ \text{the s/m is undamped}\end{array}\right.$$

where, $\omega_d = \omega_n\sqrt{1-\varepsilon^2}$

Here ω_d is called damped frequency of oscillation of the s/m & its unit is Rad/sec.

Rep 18/10

Response of undamped IInd order s/m for unit

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\varepsilon\omega_n s+\omega_n^2}$$

undamped s/m $\varepsilon = 0$.

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+\omega_n^2}$$

$$C(s) = \frac{\omega_n^2}{s^2+\omega_n^2} \times R(s)$$

$R(s) = i/p = \frac{1}{s}$

$$C(s) = \frac{\omega_n^2}{s(s^2+\omega_n^2)} \longrightarrow ①$$

$$\frac{\omega_n^2}{s(s^2+\omega_n^2)} = \frac{A}{s} + \frac{Bs+C}{s^2+\omega_n^2} \longrightarrow ②$$

② $\times s(s^2+\omega_n^2) \Rightarrow$

$$\omega_n^2 = A(s^2+\omega_n^2) + (Bs+C)s$$

$$\omega_n^2 = As^2 + A\omega_n^2 + Bs^2 + Cs$$

constant $\Rightarrow \omega_n^2 = A\omega_n^2$ $\boxed{A = 1}$

coefficient of $s \Rightarrow 0 = Cs$ ie, $\boxed{C = 0}$.

coefficient of $s^2 \Rightarrow A + B = 0$.

$A = 1$. $1 + B = 0$.

$\boxed{B = -1}$

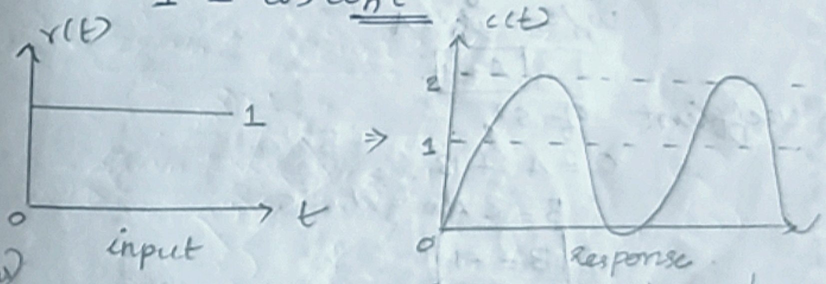
$$c(t) = L^{-1}\{C(s)\}$$

$$\frac{\omega_n^2}{s(s^2+\omega_n^2)} = \frac{1}{s} + \frac{-1\times s+0}{s^2+\omega_n^2}$$

$$c(t) = L^{-1}\left\{\frac{1}{s} - \frac{s+0}{s^2+\omega_n^2}\right\}$$

$$c(t) = L^{-1}\left\{\frac{1}{s}\right\} - L^{-1}\left\{\frac{s+0}{s^2+\omega_n^2}\right\} \quad \boxed{\varepsilon = 0}$$

$$\underline{\underline{c(t) = 1 - \cos\omega_n t}}$$



(in syllabus) imp

Response of undamped second order s/m for unit step input.

Response of critically damped IInd order s/m for unit step input

$$TF = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\varepsilon\omega_n s+\omega_n^2}$$

critically damped $(\varepsilon = 1)$

$$C(s) = \frac{\omega_n^2}{s^2+2\omega_n s+\omega_n^2} \times R(s)$$

input is unit step

$$C(s) = \frac{\omega_n^2}{s(s^2+2\omega_n s+\omega_n^2)} \longrightarrow (a)$$

$$\omega_n^2 = A(s+\omega_n)^2 + Bs(s+\omega_n) + Cs$$

$$\omega_n^2 = As^2+2As\omega_n + A\omega_n^2 + Bs^2 + Bs\omega_n + Cs$$

$$s^0 \Rightarrow \omega_n^2 = A\omega_n^2$$

$$A = \frac{\omega_n^2}{\omega_n^2} = 1.$$

$$\Rightarrow 0 = A+B$$

$$1+B$$

$$\boxed{B=-1}$$

$$\Rightarrow 0 = 2A\omega_n + B\omega_n + C$$

$$= 2\omega_n - \omega_n + C$$

$$\boxed{C=-\omega_n}$$

$$C(s) = \frac{A}{s} + \frac{B}{s+\omega_n} + \frac{C}{(s+\omega_n)^2}$$

$$= \frac{1}{s} - \frac{1}{s+\omega_n} - \frac{\omega_n}{(s+\omega_n)^2}$$

$$c(t) = L^{-1}\{C(s)\}$$

$$= L^{-1}\left\{\frac{1}{s} - \frac{1}{s+\omega_n} - \frac{\omega_n}{(s+\omega_n)^2}\right\}$$

$$\frac{1}{(s+a)^n} = t^{n-1}e^{-at}$$

$$\frac{1}{s+a}$$

$$= 1 - e^{-\omega_n t} - \omega_n t e^{-\omega_n t}$$

$$c(t) = 1 - e^{-\omega_n t}(1+\omega_n t)$$

Response of under damped II order system for unit step input.

IMP.

$$T.F = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\varepsilon\omega_n s+\omega_n^2}$$

Here, It is under damped system $0 < \varepsilon < 1$ and Root of the denomenator (characteristic eqn) are complex conjugate.

The roots of the denomenator are $s = -\varepsilon\omega_n \pm \omega_n\sqrt{\varepsilon^2-1}$

$\varepsilon < 1$. ε^2 is also less than 1. So, $1-\varepsilon^2$ is always +ve.

$$s = -\varepsilon\omega_n \pm \omega_n\sqrt{(-1)(1-\varepsilon^2)} = -\varepsilon\omega_n \pm j\omega_n\sqrt{1-\varepsilon^2}$$

The damped frequency oscillation,

$$\omega_d = \omega_n\sqrt{1-\varepsilon^2}$$

$$s = -\varepsilon\omega_n \pm j\omega_d.$$

$$C(s) = \frac{\omega_n^2}{s(s^2+2\varepsilon\omega_n s+\omega_n^2)}$$

$r(t) = 1$

$R(s) = 1/s.$

Taking partial fractions,

$$= \frac{A}{s} + \frac{Bs+C}{s^2+2\varepsilon\omega_n s+\omega_n^2}. \longrightarrow (a)$$

$$\omega_n^2 = A(s+\omega_n)^2 + Bs(s+\omega_n) + Cs$$

$$\omega_n^2 = As^2+2As\omega_n + A\omega_n^2 + Bs^2 + Bs\omega_n + Cs$$

coeff of $s \Rightarrow \omega_n^2 = A\omega_n^2$ $\boxed{A=1}$

$$0 = A+B$$

$$1+B \quad \boxed{B=-1}$$

$$0 = 2\omega_n + C$$

$$\boxed{C = -2\varepsilon\omega_n}$$

$c(t) = L^{-1}\left\{ c(s) \right\}$ $\boxed{c(t)=L^{-1}\{C(S)\}}$

$(\varepsilon^2 \omega_n^2)$ is adding and substrating to the IInd term,

$$\frac{1}{s} - \frac{s+2\varepsilon\omega_n}{s^2+2\varepsilon\omega_n s+\omega_n^2} \qquad \frac{1}{s} - \frac{s+2\varepsilon\omega_n}{s^2+2\varepsilon\omega_n}$$

$$\frac{1}{s} - \frac{s+2\varepsilon\omega_n}{s^2+2\varepsilon\omega_n s+\omega_n^2+\varepsilon^2\omega_n^2-\varepsilon^2\omega_n^2}$$

$$\frac{1}{s} - \frac{s+2\varepsilon\omega_n}{\underbrace{(s^2+2\varepsilon\omega_n s+\varepsilon^2\omega_n^2)}_{(a+b)^2}+\underbrace{\omega_n^2-\varepsilon^2\omega_n^2}_{\substack{\omega_n^2(1-\varepsilon^2)\\ \omega d^2}}}$$

$$\frac{1}{s} - \frac{s+2\varepsilon\omega_n}{(s+\varepsilon\omega_n^2)+\omega d^2}.$$

$$C(S) = \underset{①}{\frac{1}{s}} - \left\{ \underset{②}{\frac{s+\varepsilon\omega_n}{(s+\varepsilon\omega_n)^2+\omega_d^2}} + \underset{③}{\frac{\varepsilon\omega_n}{(s+\varepsilon\omega_n)^2+\omega d^2}} \right\}$$

3rd term ω_d - x ce ÷ $c(t)=L^{-1}\{C(S)\}^3$

$$= 1 - \left(e^{-\varepsilon\omega_n t}\cos\omega_d t + \frac{\varepsilon\omega_n}{(s+\varepsilon\omega_n)^2+\omega d^2}\right)$$

multiply & divide ωd $\longrightarrow$ $= \dfrac{\varepsilon\omega_n}{(s+\varepsilon\omega_n)^2+\omega d^2} = \dfrac{\varepsilon\omega_n\,\omega d}{\omega_d\{(1+\varepsilon\omega_n)^2+\omega d^2\}}$

Taking Inverse,

$$\frac{\varepsilon\omega_n}{\omega d}\sin\omega_d t\, e^{-\varepsilon\omega_n t}.$$

$$= 1 - e^{-\varepsilon\omega_n t}\left(\cos\omega_d t + \frac{\varepsilon\omega_n}{\omega_n\sqrt{1-\varepsilon^2}}\sin\omega_d t.\right\}$$

Response of over damped second order s/m
for unit step input

The stantard form of closed loop TF of IInd order s/m

$$\frac{c(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\varepsilon\omega_n s+\omega_n^2}$$

For overdamped s/m $\varepsilon>1$. Here we consider s_1 & s_2 are the root of the characteristic eq^n. $(s^2+2\varepsilon\omega_n s+\omega_n^2)$

$$\therefore s_1 = -\varepsilon\omega_n - \omega_n\sqrt{\varepsilon^2-1}$$

$$s_2 = \varepsilon\omega_n + \omega_n\sqrt{\varepsilon^2-1}.$$

$$\frac{c(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\varepsilon\omega_n+\omega_n^2} = \frac{\omega_n^2}{(s+s_1)(s+s_2)}$$

for unit step input $u(t)=1$ and $R(S)=1/s$.

$$C(S) = R(S)\,\frac{\omega_n^2}{(s+s_1)(s+s_2)} = \frac{\omega_n^2}{s(s+s_1)(s+s_2)}$$

By PF.

$$C(s) = \frac{\omega_n^2}{s(s+s_1)(s+s_2)} = \frac{A}{s}+\frac{B}{s+1}+\frac{c}{s+1}.$$

Here, the constents are .

$$A = \frac{\omega_n^2}{s_1 s_2}. \qquad \boxed{A=1}$$

$$B = \frac{\omega_n^2}{-s_1(-s_1+s_2)}$$

$$\boxed{B = -\frac{\omega_D}{2\sqrt{\varepsilon^2-1}}\frac{1}{s_1}}$$

$$\boxed{C = \frac{\omega_n}{2\sqrt{\varepsilon^2-1}}\frac{1}{s_2}.}$$

$$c(t) = L^{-1}\left\{\frac{1}{s} - \frac{\omega_n}{2\sqrt{\varepsilon^2-1}}\frac{1}{s_1}\frac{1}{(s+s_1)} + \frac{\omega_n}{2\sqrt{\varepsilon^2-1}}\frac{1}{s^2}\frac{1}{(s+s_2)}\right\}$$

$$c(t) = 1 - \frac{\omega_n}{2\sqrt{\varepsilon^2-1}}\frac{1}{s_1}e^{-s_1t} + \frac{\omega_n}{2\sqrt{\varepsilon^2-1}}\frac{1}{s^2}e^{-s_2t}$$

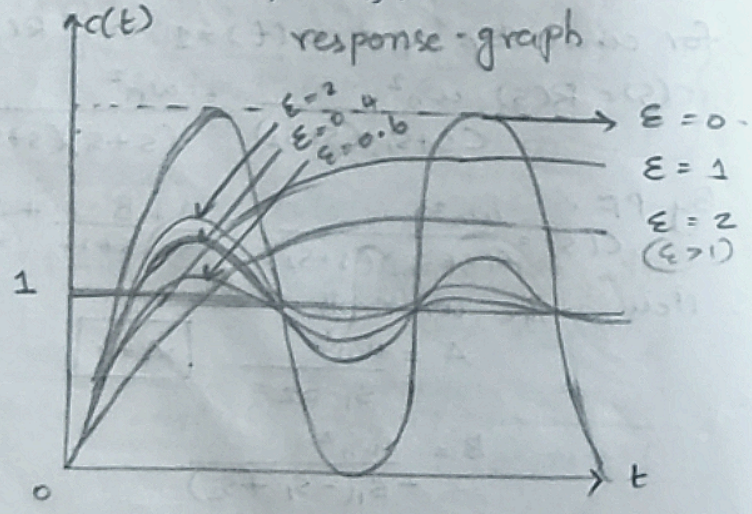
$$c(t) = 1 - \frac{\omega_n}{2\sqrt{\varepsilon^2-1}}\left(\frac{e^{-s_1t}}{s_1} - \frac{e^{-s_2t}}{s_2}\right)$$

where $s_1 = \varepsilon\omega_n - \omega_n\sqrt{\varepsilon^2-1}$

$s_2 = \varepsilon\omega_n + \omega_n\sqrt{\varepsilon^2-1}$

$\varepsilon\uparrow$ o/p $\downarrow$

response - graph



Time domain specification

Delay time (t_d)

It is the time taken for response to reach 50% of the final value. For the very I^{st} time

Rise time (t_r)

It is the time taken for response to raise from 0 to 100% of the final value.

under damped s/m → 0 to 100%.

over damped s/m → 10 to 90%.

critically damped s/m → 5 to 95%.

peak time (t_p)

It is the time taken for the response to reach the peak value.

Peak over shoot (M_p)

It is defined as the ratio of the maximum peak value to the final value, where the max-peak value is measured from final value.

Let $c(\infty)$ = final value of $c(t)$

$c(t_p)$ = maximum value of $c(t)$

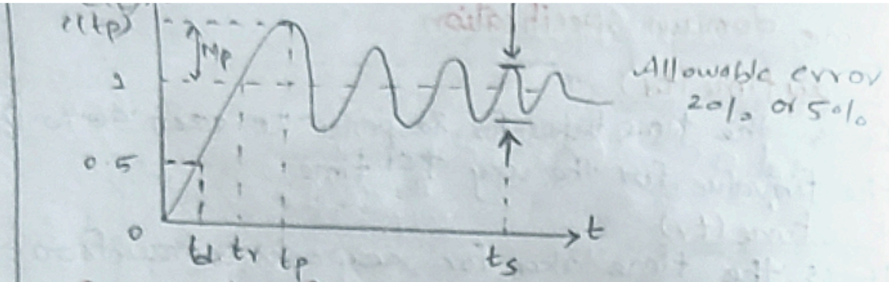
$$\text{Peak overshoot}, M_p = \frac{c(t_p) - c(\infty)}{c(\infty)}$$

% Peak overshoot

$$\%M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100.$$

Settling time (t_s)

It is defined as the time taken by the response to reach and stay with in a specified error. It is usally expressed as % of final value. The tolerable error is 2% or 5% of the final value.



Expression for Time domain specification

Rise time (tr)

Imp Response of under damped s/m,

$$c(t) = 1 - e^{-\varepsilon\omega_n t}\left\{\cos\omega_d t + \frac{\varepsilon\omega_n}{\omega_d}\sin\omega_d t\right\}$$

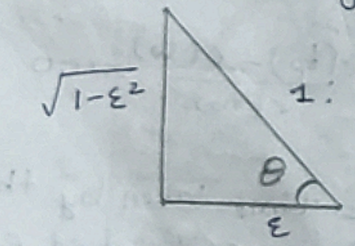
$$c(t) = 1 - e^{-\varepsilon\omega_n t}\left\{\cos\omega_d t + \frac{\varepsilon\cancel{\omega_n}\sin\omega_d t}{\cancel{\omega_n}\sqrt{1-\varepsilon^2}}\right\}$$

LCM

$$c(t) = 1 - e^{-\varepsilon\omega_n t}\left\{\frac{\sqrt{1-\varepsilon^2}\cos\omega_d t + \varepsilon\sin\omega_d t}{\sqrt{1-\varepsilon^2}}\right\}$$

$$c(t) = 1 - \frac{e^{-\varepsilon\omega_n t}}{\sqrt{1-\varepsilon^2}}\left(\sqrt{1-\varepsilon^2}\cos\omega_d t + \varepsilon\sin\omega_d t\right\}$$

consider a traingle with sides ε & $\sqrt{1-\varepsilon^2}$



$\sin\theta = \sqrt{1-\varepsilon^2}$.

$\cos\theta = \varepsilon$

$$c(t) = 1 - \frac{e^{-\varepsilon\omega_n t}}{\sqrt{1-\varepsilon^2}}\left(\sin\theta\cos\omega_d t + \cos\theta\sin\omega_d t\right)$$

$$c(t) = 1 - \frac{e^{-\varepsilon\omega_n t}}{\sqrt{1-\varepsilon^2}}\left(\sin\theta\cos\omega_d t + \cos\theta\sin\omega_d t\right)$$

$\sin A\cos B + \cos A\sin B = \sin(A+B)$

$$c(t) = 1 - \frac{e^{-\varepsilon\omega_n t}}{\sqrt{1-\varepsilon^2}}\sin(\omega_d t_r + \theta)$$

$\boxed{c(t_r) = 1}$ tr

c(t) o/p → 100% = 1

$$\boxed{c(t_r) = 1}$$

$$1 - \frac{e^{-\varepsilon\omega_n t_r}}{\sqrt{1-\varepsilon^2}}\sin(\omega_d t + \theta) = 1$$

$$\underbrace{\frac{-e^{-\varepsilon\omega_n t_r}}{\sqrt{1-\varepsilon^2}}}_{\text{1st term}}\ \overset{\text{2nd term}}{\sin(\omega_d t_r + \theta)} = 0.$$

1st ⇒ expeential term. ie $\neq 0$.

2nd ⇒ $\sin(\omega_d t + \theta) = 0$.

$\sin t = 0$,
when $t = 0, \pi, 2\pi, \ldots$

$(\omega_d t_r + \theta) = \pi$

$\omega_d t_r = \pi - \theta$.

$$t_r = \frac{\pi - \theta}{\omega_d}$$

Imp

Expression for the peak value (tp)?

Response of a 2nd order s/m,

$$c(t) = 1 - \frac{e^{-\varepsilon\omega_n t}}{\sqrt{1-\varepsilon^2}} \sin(\omega_d t + \theta)$$

for getting peak time.

$$\omega_d = \omega_n\sqrt{1-\varepsilon^2}$$

$$\frac{d}{dt} c(t) = 0.$$

$$= 0 - \frac{1}{\sqrt{1-\varepsilon^2}}\left\{ e^{-\varepsilon\omega_n t} \cos(\omega_d t + \theta) \times \omega_d + \sin(\omega_d t + \theta)\, e^{-\varepsilon\omega_n t} \times -\varepsilon\omega_n \right\}$$

$$= \frac{-e^{-\varepsilon\omega_n t}}{\sqrt{1-\varepsilon^2}}\left\{ \cos(\omega_d t + \theta)\, \omega_n\sqrt{1-\varepsilon^2} - \varepsilon\omega_n \sin \omega_d t + \theta \right\}$$

$$= \frac{-e^{-\varepsilon\omega_n t} \times \omega_n}{\sqrt{1-\varepsilon^2}}\left\{ \cos(\omega_d t+\theta)\sqrt{1-\varepsilon^2} - \varepsilon\omega_n \sin(\omega_d t + \theta) \right\}$$

put, $\sqrt{1-\varepsilon^2} = \sin\theta$

$$= \frac{e^{-\varepsilon\omega_n t} \times \omega_n}{\sqrt{1-\varepsilon^2}}\left\{ \cos\theta \sin(\omega_d t + \theta) - \sin\theta \cos(\omega_d t + \theta) \right.$$

$$= \frac{\omega_n e^{-\varepsilon\omega_n t}}{\sqrt{1-\varepsilon^2}}\left\{ \sin(\omega_d t + \cancel{\theta} - \cancel{\theta}) \right\}$$

$$= \frac{\omega_n e^{-\varepsilon\omega_n t}}{\sqrt{1-\varepsilon^2}} \sin \omega_d t$$

for getting tp.

$$\frac{d}{dt}(c_{tp}) = 0$$

$$= \frac{\omega_n e^{-\varepsilon\omega_n t_p}}{\sqrt{1-\varepsilon^2}} \times \sin \omega_d t_p = 0.$$

here exponential term $\neq 0$

$\sin \omega_d t_p = 0.$

Since, $\omega_d t_p$ have the values

$\Rightarrow 0, \pi, 2\pi \ldots$

$$\omega_d t_p = \pi$$

$$t_p = \frac{\pi}{\omega_d}$$

Peak time value.

Expression for peak over shoot (Mp)?

$$\text{Percentage overshoot} = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100.$$

$$c(t) = 1 - \frac{e^{-\varepsilon\omega_n t}}{\sqrt{1-\varepsilon^2}} \sin(\omega_d t + \theta)$$

$$c(t_p) = 1 - \frac{e^{-\varepsilon\omega_n t_p}}{\sqrt{1-\varepsilon^2}} \sin(\omega_d t_p + \theta)$$

we have $t_p = \frac{\pi}{\omega_d}$

$$c(t_p) = 1 - \frac{e^{-\varepsilon\omega_n \pi/\omega_d}}{\sqrt{1-\varepsilon^2}} \times \sin\left(\cancel{\omega_d} * \frac{\pi}{\cancel{\omega_d}} + \theta\right)$$

$$c(t_p) = 1 - \frac{e^{-\frac{\varepsilon\cancel{\omega_n}\pi}{\cancel{\omega_n}\sqrt{1-\varepsilon^2}}}}{\sqrt{1-\varepsilon^2}} \sin(\pi + \theta)$$

$$= 1 - \frac{e^{-\frac{\varepsilon\pi}{\sqrt{1-\varepsilon^2}}}}{\sqrt{1-\varepsilon^2}} \times -\sin\theta.$$

$$c(t_p) = \frac{1 + e^{-\varepsilon\pi/\sqrt{1-\varepsilon^2}}}{\sqrt{1-\varepsilon^2}} \sin\theta .$$

$$\boxed{e^{-\infty} = 0}$$

$$c(\infty) = 1 - \frac{e^{-\varepsilon\omega_n \times \infty}}{\sqrt{1-\varepsilon^2}} \sin(\omega_d \times \infty + \theta)$$

$$c(\infty) = 1 .$$

$$M_p = \frac{\dfrac{1+e^{-\varepsilon\pi/\sqrt{1-\varepsilon^2}}}{\sqrt{1-\varepsilon^2}} \sin\theta - 1}{1} \times 100 .$$

$\sin\theta = \sqrt{1-\varepsilon^2}$

$$\text{percentage overshoot} = \frac{\dfrac{1+e^{-\varepsilon\pi/\sqrt{1-\varepsilon^2}}}{\sqrt{1-\varepsilon^2}} \sqrt{1-\varepsilon^2} - \cancel{1}}{1} \times 100 .$$

$$M_p = e^{-\varepsilon\pi/\sqrt{1-\varepsilon^2}} \times 100$$

Expression for setting time (t_s)

The response of a IInd slm has 2 compoents

1) decaying expoential compoent.

ie, $\dfrac{e^{-\varepsilon\omega_n t}}{\sqrt{1-\varepsilon^2}}$

2) sinusoidal compoents $\sin(\omega_d t + \theta)$

Here, The decaying expoential compoent reduces the oscillations produced by sinusoidal compoent. Hence, the setting time is decided by the expoential compoent

case. 1

2% tolerance error band, at $t = t_s$ & equat that in to tolerance level

ie, $\dfrac{e^{-\varepsilon\omega_n t_s}}{\sqrt{1-\varepsilon^2}} = 0.02$ consider $\varepsilon \approx 0$

For least value of ε, $e^{-\varepsilon\omega_n t_s} = 0.02$ → eliminating e term by taking log by ln.

on taking natural loagrithm in bothsides we get,

$$-\varepsilon\omega_n t_s = \ln(0.02)$$

$$e^{-\varepsilon\omega_n t_s} = 0.2$$

$$\ln e^{-\varepsilon\omega_n t_s} = \ln(0.02)$$

$$-\varepsilon\omega_n t_s = \ln(0.2)$$

$$t_s = \frac{\ln(0.02)}{-\varepsilon\omega_n} = \frac{-4}{\varepsilon\omega_n}$$

2% tolerance level.

$$\boxed{t_s = \frac{4}{\varepsilon\omega_n}}$$

For a IInd order slm the time constant $T = \dfrac{1}{\varepsilon\omega_n}$

$\therefore$ Setting time $t_s = \underline{4T}$.

case - 2 For 5% error

$$e^{-\varepsilon\omega_n t_s} = 0.05$$

on taking natural loagrithm we get,

$-\xi\omega_n t_s = \ln(0.05)$

$\Rightarrow -\xi\omega_n t_s = -3$

$\Rightarrow t_s = \frac{3}{\xi\omega_n}$

Setting time, $\boxed{t_s = \frac{3}{\xi\omega_n} = 3T}$ (for 5% error) $\rightarrow$ ⓐ

In general for a specified percentage error, settling time can be evaluated using eq.n ⓑ

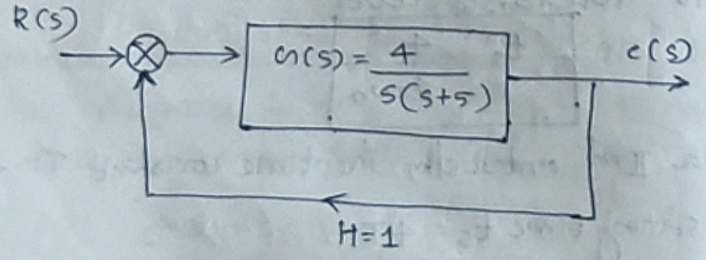
$\therefore$ setting time, $t_s = \frac{\ln(\%\text{error})}{\xi\omega_n} = \frac{\ln(\text{error})}{T}$ $\rightarrow$ ⓑ

Q obtain the response of unity feedback whoes open loop TF is $G(s) = \frac{4}{s(s+5)}$ and when the input is unit step

Sol^n.

open loop TF = TF with out feedback.

closed loop TF = TF with feedback.



$R(s) = \frac{1}{s}$ (step input)

$C(s) = ?$

$\frac{C(s)}{R(s)}$ = closed loop TF $= \frac{G}{1+GH}$

$= \frac{\frac{4}{s(s+5)}}{1+\frac{4}{s(s+5)} \times 1} = \frac{\frac{4}{s(s+5)}}{\frac{s(s+5)+4}{s(s+5)}} = \frac{\frac{x}{y}}{\frac{a}{b}}$

$= \frac{x}{y} \times \frac{b}{a} \Rightarrow \frac{4}{\cancel{s(s+5)}} \times \frac{\cancel{s(s+5)}}{s^2+5s+4}$

$\frac{C(s)}{R(s)} = \frac{4}{s^2+5s+4}$ $\Rightarrow$ Take the value s^2+5s+4.

$\underbrace{\text{Product} = 4, \quad \text{Sum} = 5}$ numbers are 1 & 4

$s^2+5s+4 = (s+1)(s+4)$

$\frac{C(s)}{R(s)} = \frac{4}{(s+1)(s+4)}$

$C(s) = \frac{4}{\cancel{s}(s+1)(s+4)} \times R(s).$

$C(s) = \frac{4}{(s+1)(s+4)} \times \frac{1}{s}$

Taking partial Fraction method,

$\frac{4}{s(s+1)(s+4)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}.$

$4 = A(s+1)(s+4) + B(s)(s+4) + C(s+1)(s)$

$s=4$ $A = \frac{4}{s(s+1)(s+4)} \times \frac{1}{s} \Big|_{s=1} \Rightarrow \underline{\underline{1}}$

$B = \frac{4}{s(s+1)(s+4)} \times \frac{1}{s+1} \Big|_{s=-1} \Rightarrow \underline{\underline{\frac{4}{-3}}}$

$$C = \frac{4}{s(s+1)(s+4)} \times \frac{1}{s+4}\Bigg|_{s=-4}$$

$$= \frac{4}{-4(-3)(0)} \times \frac{1}{-4+4} = \frac{1}{3}$$

The Time domain response $c(t)$ is obtained by taking $L^{-1}(C(s))$

$$c(t) = L^{-1}\{C(s)\}$$

$$= L^{-1}\left\{\frac{1}{s} - \frac{4}{3}\frac{1}{s+1} + \frac{1}{3}\frac{1}{s+4}\right\}$$

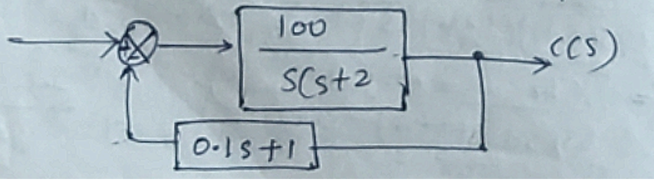
$$= 1 - \frac{4}{3}e^{-t} + \frac{1}{3}e^{-4t}$$

$$= 1 - \frac{1}{3}\left[4e^{-t} - e^{-4t}\right]$$

∴ Response of unity feedback s/m

$$c(t) = 1 - \frac{1}{3}\left[4e^{-4t} - e^{-4t}\right]$$

A positional control s/m with velocity feedback shown in figure. find what is the response of the s/m for unit step



for closed loop s/m

$$\frac{C(S)}{R(S)} = \frac{G}{1+GH} \qquad G(S) = \frac{100}{S(S+2)}, \; H(S) = 0.1S+1$$

$$\frac{C(S)}{R(S)} = \frac{\frac{100}{(S+2)S}}{1+\frac{100}{S(S+2)}(0.1S+1)} = \frac{\frac{100}{S(S+2)}}{\frac{S(S+2)+100(0.1S+1)}{S(S+2)}} = \frac{100}{S^2+2S+10S+100}$$

$$= \frac{100}{S^2+12S+100}$$

$$S_1, S_2 = \frac{-12 \pm \sqrt{144-400}}{2} = \frac{-12 \pm j16}{2} = -6 \pm j8$$

$$C(S) = R(S) \times \frac{100}{S^2+12S+100} \qquad R(S) = 1/S$$

$$C(S) = \frac{100}{S(S^2+12S+100)} = \frac{A}{S} + \frac{BS+C}{S^2+12S+100}$$

$$100 = AS^2 + 12AS + 100A + BS^2 + CS$$

$$0 = A + B \Rightarrow B = -A = -1$$

$$0 = 12A + C \quad \therefore C = -12A = -12$$

$$C(S) = \frac{1}{S} + \frac{-S-12}{S^2+12S+100}$$

$$= \frac{1}{S} - \frac{S+12}{S^2+12S+36+64}$$

$$= \frac{1}{S} - \frac{S+6+6}{(S+6)^2+8^2}$$

$$= \frac{1}{S} - \frac{S+6}{(S+6)^2+8^2} - \frac{6}{8}\frac{8}{(S+6)^2+8^2}$$

$$c(t) = L^{-1}\{C(S)\} = L^{-1}\left\{\frac{1}{S} - \frac{S+6}{(S+6)^2+8^2} - \frac{6}{8}\frac{8}{(S+6)^2+8^2}\right\}$$

$$= 1 - e^{-6t}\cos 8t - \frac{6}{8}e^{-6t}\sin 8t = 1 - e^{-6t}\left[\frac{6}{8}\sin 8t + \cos 8t\right]$$

Type no. of control s/m

The Type no is specified for loop TF function $G(S) \cdot H(S)$. The no of poles of the loop TF lying @ the origin decides the type no of the s/m

$$G(S)H(S) = K\frac{P(S)}{Q(S)} = \frac{K(S+Z_1)(S+Z_2)(S+Z_3)\ldots}{S^N(S+P_1)(S+P_2)(S+P_3)\ldots}$$

where $Z_1, Z_2, Z_3 \ldots$ are zeros of TF

$P_1, P_2, P_3 \ldots$ are poles of TF

K = constant

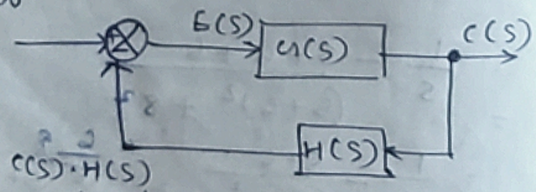
N = no of poles at the origin

The value
If N = 0 then s/m is type - 0 s/m
If N = 1 " " " – 1 s/m
If N = 2 " " " – 2 s/m
If N = 3 " " " – 3 s/m
and so on

IMP

Steady state error

It Is the value of error signal e(t) when $t \to \infty$



consider a closed loop s/m,

$R(S)$ = Input signal

$E(S)$ = Error signal

$C(S) \cdot H(S)$ = feedback signal

$C(S)$ = Response

The error signal $E(S) = R(S) - C(S)H(S)$

The o/p signal $C(S) = E(S)G(S)$

sub value of $C(S)$ in $E(S)$

$$E(S) = R(S) - [E(S)G(S)]H(S)$$

$$E(S) + [E(S)G(S)]H(S) = R(S)$$

$$E(S)[1 + G(S)H(S)] = R(S)$$

$$\therefore E(S) = \frac{R(S)}{1 + G(S)H(S)}$$

let e(t) = error signal in time domain

$$e(t) = L^{-1}\{E(S)\}$$

$$e(t) = L^{-1}\left\{\frac{R(S)}{1+G(S)H(S)}\right\}$$

(SUB Q) let e_{ss} = steady state error.

The steady state error is defined as the value of e(t) when $t \to \infty$.

$$e_{ss} = \underset{t \to \infty}{Lt}\, e(t)$$

The final value theorm of LT states that,

If, $F(S) = L\{f(t)\}$ then,

$$\underset{t \to \infty}{Lt} f(t) = \underset{s \to 0}{Lt} SF(S)$$

using final value Theorm,

The steady state error

$$e_{ss} = \underset{t\to\infty}{Lt}\; e(t) = \underset{s\to 0}{Lt}\; sE(s)$$

$$= \underset{s\to 0}{Lt}\; \frac{sR(s)}{1+G(s)H(s)}$$

Static error constants

when a control s/m is excited with standard i/p signal, the steady state error may be zero, constant or Infinity The value of steady state error depends on the type number and i/p signal.

$s^0 \Rightarrow$ Type 0 Positional error constent, $K_p = \underset{s\to 0}{Lt}\; G(s)\,H(s)$

$s^1 \Rightarrow$ Type 1 velocity error constant, $K_v = \underset{s\to 0}{Lt}\; sG(s)\,H(s)$

$s^2 \Rightarrow$ Type 2 Acceleration error constent $K_a = \underset{s\to 0}{Lt}\; s^2 G(s)H(s)$

The K_p, K_v, K_a are in general called static error constants.

Q, For a unity Feedback control s/m the open loop TF, $G(s) = \frac{10(s+2)}{s^2(s+1)}$. find, the position, velocity and acceleration error constants,

Sol^n

$$G(s) = \frac{10(s+2)}{s^2(s+1)}$$

$K_p = ?$

$K_v = ?$

$K_a = ?$

unity feedback $H(s) = 1$

$$K_p = \underset{s\to 0}{Lt}\; G(s)H(s)$$

$$K_v = \underset{s\to 0}{Lt}\; sG(s)H(s)$$

$$K_a = \underset{s\to 0}{Lt}\; s^2 G(s)\,H(s)$$

K_p. $\underset{s\to 0}{Lt}\; \frac{10(s+2)}{s^2(s+1)} = \frac{20}{0^2(0+1)} = \infty$

K_v $\underset{s\to 0}{Lt}\; \cancel{s}\; \frac{10(s+2)}{\cancel{s^2}(s+1)} = \frac{(s+2)10}{s(s+1)}\;{}_{s\to 0} = \frac{20}{0} = \infty$

K_a $\underset{s\to 0}{Lt}\; \cancel{s^2}\; \frac{10(s+2)}{\cancel{s^2}(s+1)} = \frac{10(s+2)}{(s+1)} = 20$

28/10/22

Q For servo mechanisms with open loop TF given below explian what type of input signal gives rise to a constant steady state error and calculate their values. consider unity feeback 1. $H(s) = 1$

a) $G(s) = \frac{20(s+2)}{s(s+1)(s+3)}$ b) $G(s) = \frac{10}{(s+2)(s+3)}$

c) $G(s) = \frac{10}{s^2(s+1)(s+2)}$.

Sol^n

Input signal	Type no. of s/m.			
	0	1	2	3
unit step	$\frac{1}{1+K_p}$	0	0	0
unit Ramp	∞	$\frac{1}{K_v}$	0	0
unit parabolic	∞	∞	$\frac{1}{K_a}$	0

a) $G(s) = \frac{20(s+2)}{s(s+1)(s+3)}$

Consider $H(s) = 1$

$G(s)H(s) = \frac{20(s+2)}{\boxed{s}(s+1)(s+3)}$ $s^1 =$ Type 1

Type of system = 1.

we want e_{ss} = constant @ unit Ramp input.

$e_{ss} = \frac{1}{K_v}$

$K_v = \lim_{s \to 0} s\,G(s)\,H(s)$.

$= \lim_{s \to 0} \cancel{s} \frac{20(s+2)}{\cancel{s}(s+1)(s+3)}$ $\boxed{2 \times 2}$ $= \frac{40}{3}$

1×4

$e_{ss} = \frac{1}{K_v} = \frac{1}{\frac{40}{3}} = \frac{3}{40}$

b) $G(s) = \frac{10}{(s+2)(s+3)}$

$H(s) = 1$

$G(s)H(s) = \frac{10}{(s+2)(s+3)}$ $s^0 \Rightarrow$ Type 0.

e_{ss} = constant when i/p is unit step

$e_{ss} = \frac{1}{1+K_p}$

$K_p = \frac{1}{1+}$ $\frac{1}{+10} = \frac{1}{6+10} = \frac{1}{\frac{16}{6}}$

$K_p = \lim_{s \to 0} G(s)H(s)$.

$= \lim_{s \to 0} \frac{10}{(s+2)(s+3)} = \frac{10}{6}$

$e_{ss} = \frac{6}{16}$

c) $G(s) = \frac{10}{s^2(s+1)(s+2)}$

$H(s) = 1$ $s^2 =$ Type 2

e_{ss} = constant @ unit parabolic i/p.

$e_{ss} = \frac{1}{K_a}$

$K_a = \lim_{s \to 0} s^2\,G(s)\,H(s)$.

$\lim_{s \to 0} \cancel{s^2} \frac{10}{\cancel{s^2}(s+1)(s+2)}$

$\frac{10}{(0+1)(0+2)} = \frac{10}{2}$

$e_{ss} = \frac{1}{K_a} = \frac{1}{\frac{10}{2}} = \frac{2}{10} = 1/5$

3um Controllers

A controller is a device introduced in the s/m to modify the error signal and to produce a control signal.

The following six basic control actions are Very common among Industrial analog controllers

1) Two-position or ON-OFF control action
2) proportional control action
3) Integral control action
4) proportional plus - integral control action
5) " " - derivative " "
6) " " - integral plus - derivative control action

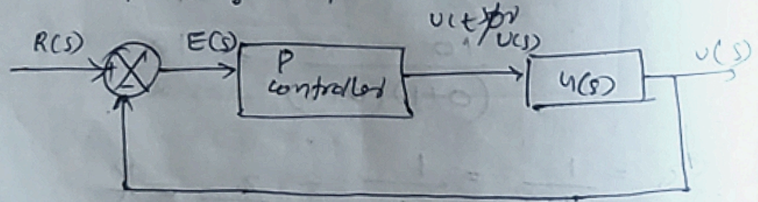
proportional controller (P- controller)

The P controller is a device that produce a control signal, $u(t) \propto$ to the i/p error signal $e(t)$.

In p controller, $u(t) \propto e(t)$

$\therefore u(t) = k_p e(t)$

$k_p = \propto$ gain / constant



Taking LT of equation

$U(S) = K_p E(S)$

T.F of P controller $\frac{U(S)}{E(S)} = k_p$

Advantage

- P controller amplifies the error signal by amount of k_p
- It ↑ loop gain
- Improve steady state accuracy
- Improve disturbance signal rejection
- Improves relative stability
- decreses sensitivity.
- constant steady state error } Limitations

Integral controller (I - controller)

The Integral controller is a device that produces a control signal $u(t)$ which is proportional to Integral of the i/p error signal $e(t)$.

In I. controller

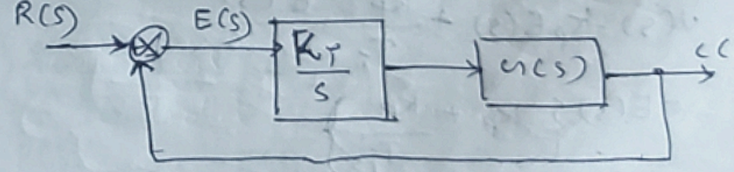
$$u(t) \propto \int e(t)dt \Rightarrow u(t) = K_I \int e(t)dt \longrightarrow ⓐ$$

$k_I \Rightarrow$ integral gain / constant.

on taking LT of eqn ⓐ with zero Initial condition

$U(S) = K_I \frac{E(S)}{S}$

TF of I - controller, $\frac{U(S)}{E(S)} = \frac{K_I}{S}$



Features:-

1) It redues the steady state error hence It is known as automatic reset.
2) It create oscillatory response.
3) It ~~s/m~~ make s/m is unstable

Proportional plus Integral controller (P-I)

It produces an o/p signal consisting of 2 terms: one α error signal and other α Integral of error s/g.

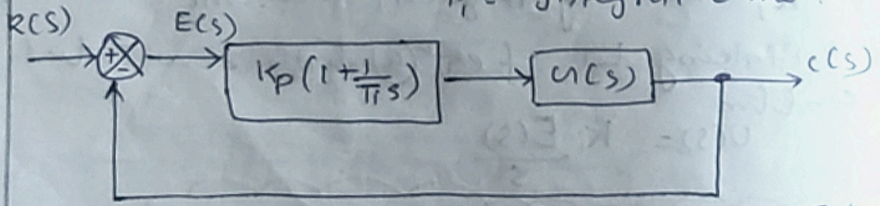
In P.I controller,

$$u(t) \propto \left[e(t) + \int e(t)dt\right];$$

$$\therefore u(t) = K_p e(t) + \frac{K_p}{T_i}\int e(t)dt \longrightarrow ⓐ$$

K_p = proportional gain

T_i = Integral time



on taking LT of eqⁿ ⓐ with zero Initial conditions we get,

$$U(S) = K_p E(S) + \frac{K_p}{T_i}\frac{E(S)}{S}$$

$$= E(S)\left\{K_p + \frac{K_p}{T_i s}\right\}$$

$$\frac{U(S)}{E(S)} = K_p + \frac{K_p}{T_{is}} = K_p\left(1 + \frac{1}{T_{is}}\right)$$

$$\boxed{\text{TF of PI controler } \frac{U(S)}{E(S)} = K_p\left(1 + \frac{1}{T_i s}\right)}$$

features:-

1) It Incresses the loop gain ~~and~~
2) It makes the s/m less sensitive to various ~~variation~~ s/m parameter (sensitivity ↑)
3) It redues the steady state error

proportional plus derivative controller (P-D)

The proportional plus derivative produces an o/p signal consisting of two terms: one proportional to error signal & other α to the derivative of error signal

In P.D controller, $u(t) \propto \left[e(t) + \frac{d}{dt}e(t)\right]$

$$\therefore u(t) = K_p e(t) + K_p T_d \frac{d}{dt}e(t) \longrightarrow ⓐ$$

K_p = proportional gain

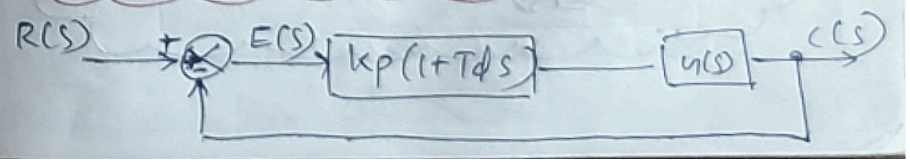
T_d = derivative time

LT of eqⁿ ⓐ with zero IC,

$$U(S) = K_p E(S) + K_p T_d S E(S).$$

$$U(S) = E(S)\left\{K_p + K_p T_d s\right\}$$

$$\boxed{\frac{U(S)}{E(S)} = K_p(1 + T_d s) \text{ i.e, TF}}$$

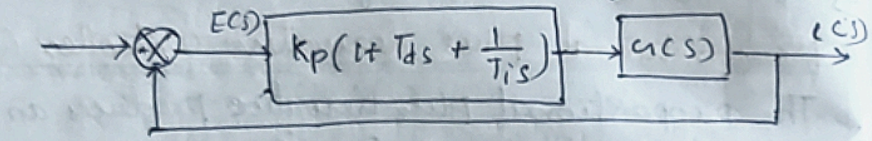


Features

- It is effective for Transient period. (PID)

Proportional plus Integral plus derivative contr

It produces an o/p signal consisting of a three terms: one $\propto$ error signal, another one $\propto$ to integral of error signal and the third one $\propto$ to derivative of error signal



PID controller

$$u(t) \propto \left[e(t)+\int e(t)dt \frac{d}{dt}e(t)\right]$$

$$\therefore u(t)=K_p e(t)+\frac{K_p}{T_i}\int e(t)dt+K_pT_d\frac{d}{dt}e(t) \longrightarrow ⓐ$$

K_p = proportional gain

T_i = Integral time

T_d = Derivative time

on taking LT of eq ⓐ with zero Ic,

$$U(S)=K_pE(S)+\frac{K_p}{T_i}\frac{E(S)}{s}+K_pT_dsE(S) \longrightarrow ⓑ$$

TF of PID

$$\boxed{\frac{U(S)}{E(S)}=K_p\left(1+T_ds+\frac{1}{T_is}\right)} \longrightarrow ⓒ$$

MODULE-3

FREQUENCY RESPONSE ANALYSIS

The sinusoidal TF is the frequency domain representation of the s/m, & so it is called frequency domain TF.

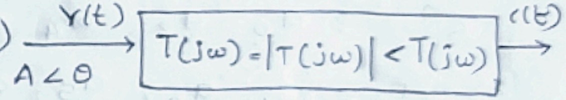
$$T(j\omega)=|T(j\omega)|\angle T(j\omega)$$

$$T(s)=T(j\omega)$$

where $|T(j\omega)|$ = mag of $T(j\omega)$

$\angle T(j\omega)$ = phase of $T(j\omega)$

$r(t)=A\sin(\omega t+\theta)$



$c(t)=B\angle\phi$

where $B=A\times|T(j\omega)|$

$\phi=\theta+\angle T(j\omega)$

s/m with sinusoidal TF $T(j\omega)$

Imp

ROUTH HURWITZ CRITERION.

It is used for the determination of stability of the system. The Ist step in analysing ~~is~~ to examine its characteristic eqn.

step-1

find characteristic equation (CE)

2) check whether all the coefficients of the C.E

is positive or not. If all are positive the s/m may be stable. otherwise the s/m is unstable.

B). If all the coeffients are +ve move to - R.H table.

4). If all the ~~all~~ elements of the Ist column of the Routh are a +ve. then the s/m - is stable.

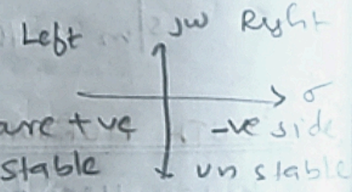
R.H Table

consider the CE is

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n = 0.$$

where $a_0 > 0$;

s^n	:	a_0	a_2	a_4	a_6	a_8 .....
s^{n-1}	:	a_1	a_3	a_5	a_7	a_9 .....
s^{n-2}	:	b_0	b_1	b_2	b_3	b_4 .....
s^{n-3}	:	c_0	c_1	c_2	c_3	c_4 .....
s^1	:	g_0				
s^0	:	h_0.				



Here, $b_0 = \dfrac{a_1 a_2 - a_0 a_3}{a_1}$.

The Routh stability criteria can be stated as follows;

ee ROUTH stability criterion:

The necessary and sufficient condition for stability is that all of the elements in the first column of the Routh array be +ve. If this condition is not met, the s/m is unstable and the no. of sign changes in the elements of the first column of the Routh array corresponds to the no. of roots of the CE in the right half of s plane

Problem.1

Q. using Routh criterion, determine the stability of the s/m represented by the CE,
$s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$. comment on the location of the roots of CE.?

sol^n: $s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$.

Power of s = order no = 4.
Hence, we have 4 roots. RH table.

s^4 :	1	18	5	
s^3 :	8	16	0.	
s^2 :	$\frac{8\times18 - 1\times16}{8} \Rightarrow 16$	$\frac{16\times5 - 18\times0}{16} \Rightarrow 5$	0	
s^1 :	$\frac{16\times16 - 5\times5}{16} \Rightarrow 13.5$	0		0
s^0 :	5	0		0.

when we go through RH table all the elements of the Ist column are +ve. Hence, the s/m is stable.

Here, The no of sign changes in the Ist column = 0 hence, all the roots are lies in left side of s plane.

⓪ Q2) construct routh array and determine the stability of the s/m whose CE is $s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$. Also determine the no. of roots lying on left half of s plane. right half of s-plane. and on imaginary axis?

−ve value → Right

Sol^n

$s^6+2s^5+8s^4+12s^3+20s^2+16^s+16=0$

power of s ⇒ order of s = 6

we have 6 roots

s^6 : 1   8   20   16

s^5 : 2   ⑫   ⑯   0.

2 common outside

$\cancel{s^4}$: $\cancel{8}$ $\cancel{20}$ $\cancel{16}$ $\cancel{0}$

$\cancel{s^3}$: $\cancel{12}$ $\cancel{16}$ $\cancel{0}$ $\cancel{0}$.

s^4 : $\frac{2\times8-12}{2}$ $\frac{12\times20-8\times16}{12}$ $\frac{16\times16-0\times20}{16}$ 0

$\frac{16-12}{2}=2$ 9.3   16

240 − 128

s^6 : 1   8   20   16

s^5 : 2   12   16   0

s^4 : $\frac{8\times2-1\times12}{2}$ = ② $\frac{2\times20-1\times16}{2}$ = ⑫ $\frac{2\times16-1\times0}{}$ = ⑯   0

s^3 : $\frac{2\times12-2\times12}{2}$ = ⓪ $\frac{4\times16-16\times2}{2}$ = ⓪.   ⓪   ⓪

one compelte row of RH table is zero then we take auxilary eq^n as the row above the zero th row and take the derivative of that auxilary eq^n.

auxilary eq^n,

$2s^4+12s^2+16=0$.

$\left\{ \frac{d}{dx}x^n = nx^{n-1} \quad \frac{d}{dx}(c)=0 \right.$

derivative (Auxilary eq^n) = 0.

$2\times4s^3+12\times2s+0=0$.

$s^4 \Rightarrow 8s^3+24s=0$

s^4 : 2   12   16   0

s^3 : 8   24   0.   0

s^2 : $\frac{8\times12-2\times24}{8}$ $\frac{8\times16-0\times2}{8}$ 0   0.

s^2 : 6   16

s^1 : $\frac{6\times24-16\times8}{6}$ = $\boxed{2.66}$ 0   0   0.

s^0 : $\frac{2.66\times16-0\times6}{2.66}$ = ⑯   0   0   0.

On examining the elements of I^{st} column of Routh array it is observed that there is nosign change. The row with all zero indicates the possibility of roots on imaginary axis. Hence the s/m is limitedly or marginally stable.

The auxilarly polynomial is

$$2S^4 + 12S^2 + 16 = 0$$

Here we take $s^2 = x$.

$$\therefore 2x^2 + 12x + 16 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad b = 12,\ a = 2,\ c = 16$$

$$x = \frac{-12 \pm \sqrt{12^2 - 4 \times 2 \times 16}}{2 \times 2}$$

$$x = \frac{-12 \pm \sqrt{144 - 128}}{4}$$

$$x = \frac{-12 \pm 4}{4} \Rightarrow \frac{-12+4}{4} = \underline{\underline{-2}}.$$

$$\frac{-12-4}{4} \Rightarrow \underline{\underline{-4}}$$

$$S^2 = x.$$

$$s = \pm\sqrt{x} = \pm\sqrt{-2} \quad \text{or} \pm\sqrt{-4}$$

$$= \pm i\sqrt{2} \quad \text{and} \pm i\,2$$

Hence, 4 roots are lying on imaginary axis and the remaining two roots are lying on the left half of s-plane.

Q construct Routh array and determine the stablity of the s/m represented by a characteristic eqn. $S^5 + S^4 + 2S^3 + 2S^2 + 3S + 5 = 0$. comment on the location of the roots of characterstic Equation

Sol:

$$S^5 + S^4 + 2S^3 + 2S^2 + 3S + 5 = 0$$

$$S^5 : \quad 1 \qquad 2 \qquad 3$$

$$S^4 : \quad 1 \qquad 2 \qquad 5$$

(0 ⇒ ε for all row → AE

$$S^3 : \quad \frac{1 \times 2 - 2 \times 1}{1}\ (\varepsilon)\ (0) \qquad \frac{1 \times 3 - 5 \times 1}{1}\ (-2)$$

$$S^2 = \frac{2\varepsilon - (-2)}{\varepsilon} = \underline{\underline{\frac{2\varepsilon + 2}{\varepsilon}}} \qquad \frac{5\varepsilon - 0}{\varepsilon} = \underline{\underline{5}}$$

$$S^1 = \frac{-2\left(\frac{2\varepsilon+2}{\varepsilon}\right) - 5\varepsilon}{\frac{2\varepsilon+2}{\varepsilon}} \Rightarrow \frac{\left(\frac{-4\varepsilon - 4 - 5\varepsilon^2}{\varepsilon}\right)}{2\varepsilon + 2/\varepsilon}$$

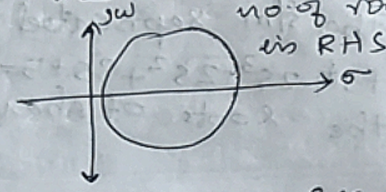
$$= \frac{-5\varepsilon^2 - 4\varepsilon - 4}{2\varepsilon + 2}.$$

$$S^0 = \underline{\underline{5}}$$

on letting $\varepsilon \to 0$, we get

s^5 : 1 2 3
s^4 : 1 2 5
s^3 : 0 −2
s^2 : ∞ 5
s^1 : −2
s^0 : 5

of RH table considering first column, except −2, all are +ve values. This −ve indicate The s/m is unstable



no. of root lies in RHS of s-plane

There are (2) no. of sign change i.e, from ∞ to −2 and −2 and to 5

∴ The s/m is unstable.

Total no. of root ~~5~~ 5, 2 root are in RHS and the remaining 3 root are lies in LHS.

Page no: 4-16.

Q) Determine the range of K for stability of unity feedback s/m whoes open loop TF is

$$G(s) = \frac{K}{S(S+1)(S+2)}$$

sol^n for finding the stability of s/m we want to construct RH table, characteristic eq^n = denomenator of CLTF.

For construction of RH table we need A.E

$$\text{closed loop T.F} = \frac{G}{1+GH} \; ; \boxed{H=1}$$

$$\therefore CLTF = \frac{G}{1+GH}.$$

$$CLTF = \frac{C(s)}{R(s)} = \frac{\frac{K}{S(S+1)(S+2)}}{1 + \frac{K}{S(S+1)(S+2)}}$$

LCM

$$= \frac{\frac{K}{\cancel{S}\cancel{(S+1)}\cancel{(S+2)}}}{\frac{S(S+1)(S+2)+K}{\cancel{S}\cancel{(S+1)}\cancel{(S+2)}}} = \frac{K}{S(S+1)(S+2)+K}$$

∴ The characteristic eq^n.

$$S(S+1)(S+2)+K=0$$

$$\therefore S(S^2+3S+2)+K=0 \Rightarrow S^3+3S^2+2S+K=0.$$

R-H table.

S^3 : 1 2
S^2 : 3 K
S^1 : $\frac{3\times2 - K\times1}{3} \Rightarrow \frac{6-K}{3}$,
S^0 : K

For the s/m to be stable there should not be any sign change in the elements of first column. Hence choose the value of K so that the first column elements are positive.

From s^0 row, the s/m to be stable, $k>0$

from s^1 row, for the s/m to be stable $\frac{6-k}{3}>0$.

For $\frac{6-k}{3}>0$, the value of K should be <6 $6>k$.

$\therefore$ The range of K for the s/m to stable, $K<6$

$0<K<6$.

Advantages of frequency Response analysis

1) The absolute and relative stability of closed loop s/m can be estimated.

2) The pratical testing of the s/m can be easily carried with available sinusoidal signal generators and precise measurement equipments.

3) The TF of complicated s/m can be determined experimentally by frequency response tests.

4) The design and parameter adjustment of the open loop TF of a s/m for specified closed loop performance is carried out more easily in frequency domain.

5) when the s/m is designed by use of the frequency response analysis, the effect of noise disturbance and parameters variations are relatively easy to visualize and incorporate corrective measures.

6) The frequency response analysis and design can be extended to certain non-linear control

Frequency domain specification

(Q unit balance)

◉ Resonant peak (M_r)

The max. value of magnitude of closed loop TF is called the resonant peak (M_r)

◉ Resonant frequency (ω_r)

The frequency @ which resonant peak occurs is called Resonant frequency (ω_r).

◉ Bandwidth (ω_b)

The BW is the range of frequency for which normalized gain of the s/m is more than -3dB. The frequency at which the gain is -3dB is called cut-off-frequency.

◉ cut-off rate.

The slope of the log-magnitude curve near the cutoff frequency is called cut-off rate. The cut-off rate indicates the ability of the s/m to distinguish the signal from noise

◉ Gain Margin, Kg

(The gain margin, Kg is defined as the value of gain, to be added to s/m, in order to bring the s/m to the verage of instability.

ω_{pc} ⇒ Phase crossover frequency

$$\text{Gain Margin } K_g = \frac{}{\left|G(j\omega_{pc})\right|}$$

The gain Margin in dB can be expressed as,

k_g in dB = $20\log k_g$

$$\Rightarrow 20\log \left| \frac{1}{|G(j\omega_{pc})|} \right|$$

k_g is given by the reciprocal of the s/m magnitude of open loop TF @ phase cross over frequency. The frequency @ which the phase of open loop TF is 180 is called phase cross over frequency, ω_{pc}

$$G = |G| \angle G$$
$$\angle G = 180.$$

◎ *Phase Margin* (γ)

(It is defined as the additional phase lag to be added at which the gain cross over frequency) in order to bring the s/m to be verge of instability. The gain cross over frequency ω_{gc} is the frequency at which the magnitude of open loop TF is unity.

(The phase margin γ, is obtained by adding 180° to the phase angle ϕ of the open loop TF @ the gain cross over frequency

$$\boxed{\text{Phase Margin, } \gamma = 180^\circ + \phi_{gc}}$$

where, $\phi_{gc} = \angle G(j\omega_{gc})$

II nd series

◎ Resonant Peak, $M_r = \dfrac{1}{2\varepsilon\sqrt{1-\varepsilon^2}}$

◎ Resonant frequency, $\omega_r = \omega_n\sqrt{1-2\varepsilon^2}$

◎ Bandwidth, $\omega_b = \omega_n\left[1-2\varepsilon^2+\sqrt{2-4\varepsilon^2+4\varepsilon^4}\right]^{1/2}$

◎ Phase margin; $\gamma = 90 - \tan^{-1}\left[\dfrac{\left[-2\varepsilon^2+\sqrt{4\varepsilon^4+1}\right]^{1/2}}{2\varepsilon}\right]$

The gain margin of II nd order system is infinite

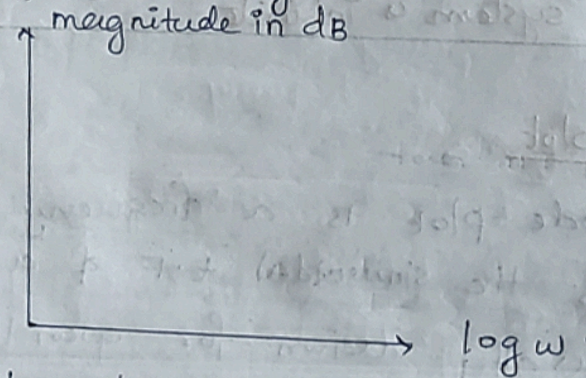
Bode plot

- The bode plot is a frequency response plot of the sinusoidal t.F of a systems.
- It is usally drawn for open loop system.
- It is used to determine frequency domain specification & stability of the s/m.

- Here we can adjust the gain of the s/m to satisfy the desired specification
- Bode plot consist of 2 plots.

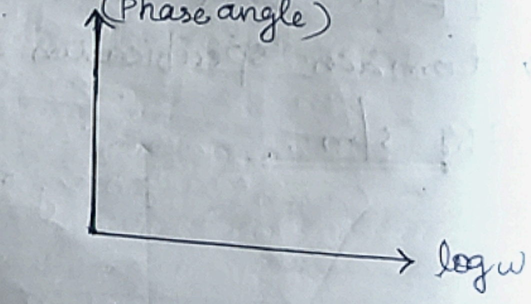
1) Magnitude plot

It is plot of magnitude of sinusoidal TF v/s log ω.



2) Phase plot

This plot ~~is a~~ ~~Bode~~ phase angle of sinusoidal TF versery log ω.

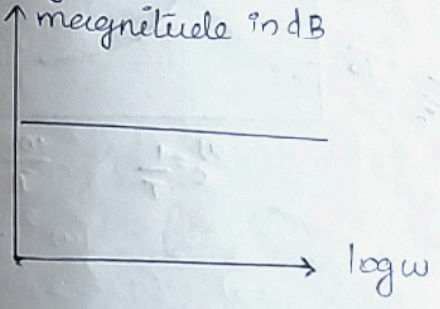


Const gain, K

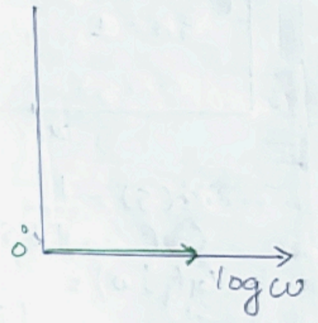
Let $G(s) = k$

$\therefore G(j\omega) = k = k \angle 0^\circ$

magnitude plot



phase plot



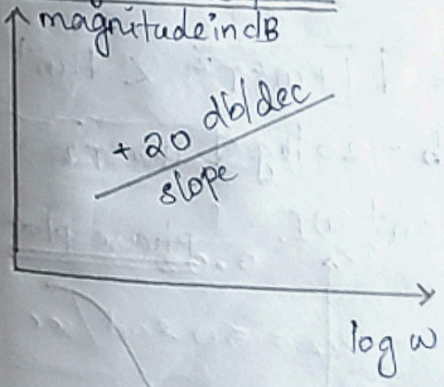
Derivative factor

Let $G(s) = KS$

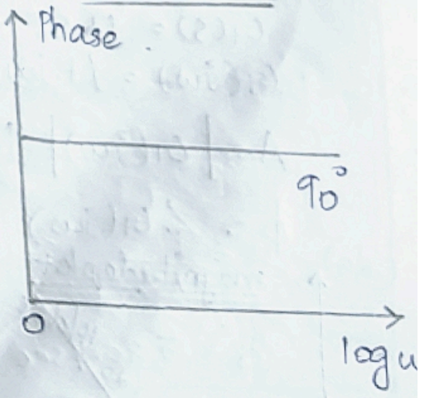
$\therefore G(j\omega) = Kj\omega$

$= K\omega \angle 90^\circ$

magnitude plot

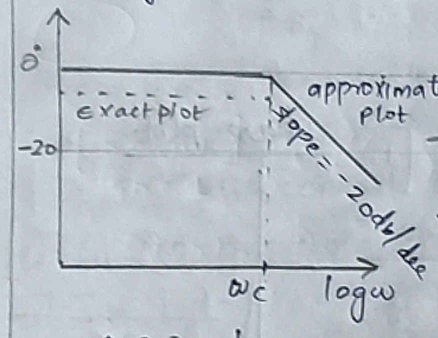


Phase plot

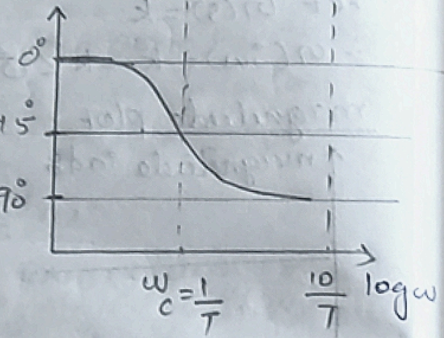


1+ST term in denominator.

magnitude plot



Phase plot



$$G(S) = \frac{1}{1+ST}$$

$$G(j\omega) = \frac{1}{1+j\omega T}$$

$$= \frac{1}{\sqrt{1+\omega^2 T^2}} \angle -\tan^{-1}\omega T.$$

corner frequency $\omega_c = \frac{1}{T}$

First order factor in the numerator

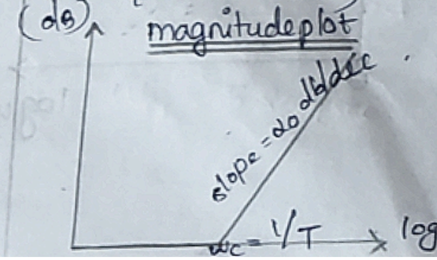
$$G(S) = 1+ST$$

$$G(j\omega) = 1+j\omega T = \sqrt{1+\omega^2T^2} \angle \tan^{-1}\omega T$$

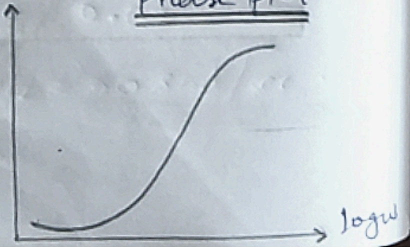
$$A = |G(j\omega)| \text{ in db} = 20\log\sqrt{1+\omega^2T^2}.$$

$$\phi = \angle G(j\omega) = \tan^{-1}\omega T.$$

magnitudeplot



phase plot



Quadratic factor in the denomenatory

$j^2 = -1$

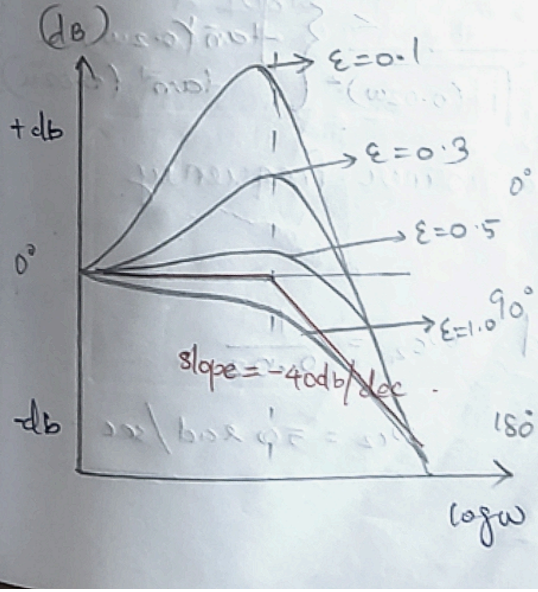
$$G(S) = \frac{\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2} = \frac{\omega_n^2}{\omega_n^2\left(\frac{S^2}{\omega_n^2}+\frac{2\xi S}{\omega_n}+1\right)}$$

$$= \frac{1}{1+\frac{2\xi S}{\omega_n}+\left(\frac{S}{\omega_n}\right)^2}$$

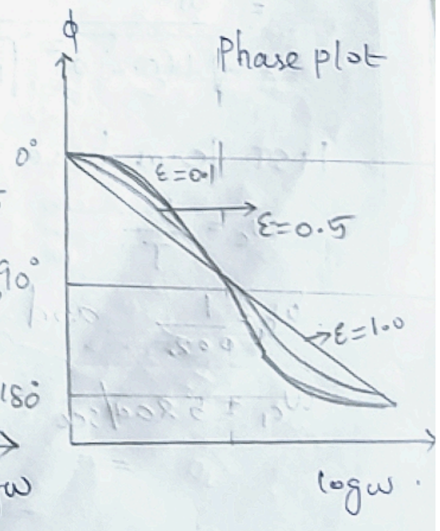
$$\therefore G(j\omega) \Rightarrow \frac{1}{1+j\frac{2\xi\omega}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2}$$

$$= \frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2+j2\xi\frac{\omega}{\omega_n}} = \frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)+\left(4\xi^2\frac{\omega^2}{\omega_n^2}\right)}} \angle \tan^{-1}\frac{\frac{2\xi\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}}$$

$$\frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)+\left(4\xi^2\frac{\omega^2}{\omega_n^2}\right)}} \angle \tan^{-1}\frac{2\frac{\xi\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}}$$



Phase plot



Q) sketch bode plot for the following T.F and determine the system gain K for the gain cross over frequency to be 5 rad/sec.

$$G(s) = \frac{Ks^2}{(1+0.2s)(1+0.02s)}$$

Sol^n

substitute $[s = j\omega$ & $K = 1]$

(s- angle 90°
s² angle 180°)

$$G(s) = \frac{Ks^2}{(1+0.2s)(1+0.02s)}$$

& K=1

$$G(j\omega) = \frac{s^2}{(1+0.2s)(1+0.02s)} = \frac{j\omega^2}{(1+0.2j\omega)(1+0.02}$$

$j^2 = -1$

$$= \frac{-\omega^2}{(1+0.2j\omega)(1+0.02j\omega)}$$

$$= \frac{\omega^2}{\sqrt{1+(0.2j\omega)^2}\sqrt{1+(0.02\omega)^2}} \angle \left\{ 180^\circ - \tan^{-1}(0.2\omega) - \tan^{-1}(0.02\omega) \right\}$$

1+sT term have corner frequency

$$\omega_c = \frac{1}{T}$$

$\omega_{c_1} = \frac{1}{0.2}$ and $\omega_{c_2} = \frac{1}{0.02}$

$\omega_{c_1} = 5$ rad/sec $\qquad \omega_{c_2} = 50$ rad/sec

magnitude plot.

Term	corner frequency rad/sec	slope db/dec	change in slope db/dec
$(j\omega)^2$	-	+40	-
(s1) $\frac{1}{1+.2j\omega}$	$\omega_{c_1} = \frac{1}{.2}$ $\omega_{c_1} = 5$ rad/sec	-20	$s_1 + s_2$ $40-20=20$
(s2) $\frac{1}{1+0.02j\omega}$	$\omega_{c_2} = \frac{1}{0.02}$ $\omega_{c_2} = 50$ rad/sec	-20	$20+{}^-20=0$.

choose 2 frequencies ω_l and ω_h.

where $\omega_{low} < \omega_{c_1}$ and

$\omega_{high} > \omega_{c_2}$

consider, $\omega_l = 0.5$ rad/sec

$\omega_h = 100$ rad/sec.

at $\omega = \omega_l$.

(0.25)

(mag) $A = 20\log|(j\omega)^2|$

$(j = -1)$ $= 20\log(\omega)^2$

$= 20\log(0.5)^2$

$= -12$ db/dec

At $\omega = \omega_{c1} = 5$ rad/sec

$A = 20 \log |(j\omega)^2|$

$= 20 \log \omega^2$

$= 20 \log (5)^2$

$= 28$ db/dec

At $\omega = \omega_{c2} = 50$ rad/sec

$A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \frac{\omega_{c2}}{\omega_{c1}}\right] + A_{(at\ \omega = \omega_{c1})}$

$A = 20 \times \log \frac{50}{5} + 28$

$A = 48$ db/dec

$50 \rightarrow 48$

$100 \rightarrow 48$

At $\omega = \omega_h$

$A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c2}}\right] + A_{(at\ \omega = \omega_{c2})}$

$A = 0 \times \log \frac{100}{56} + 48$

$A = 48$ db

Phase plot

Phase Plot

ω rad/sec	$\tan^{-1} 0.2\omega$ deg.	$\tan^{-1} 0.02\omega$ deg	$\phi = \angle G(j\omega)$ deg
0.5	5.710	0.572	≈ 174
5	45	5.710	≈ 130
10	63.43	11.309	≈ 106
50	84.28	45	≈ 50
100	87.13	63.434	≈ 30

Sol^n = 28 dB

∴ $20 \log K = -28$ db

$\log K = \frac{-28}{20}$

$K = 10^{-\left(\frac{28}{20}\right)}$

$K = 0.03981$

at $\underline{\omega = \omega_l}$

$\omega_f = 1$ rad/sec

$\omega_l = 0.5$ rad/sec $\downarrow$ 3.5 db

$A = 20 \log \left| \frac{0.75}{j\omega_l} \right|$

$(j = -1)$ $A = 20 \log \left| \frac{0.75}{-1 \times 1} \right|$

$20 \log \cdot \left(\frac{3}{4}\right) = -2.498 \approx -2.5$ db

$\underline{A@ \omega = \omega_{c_1}} \Rightarrow A = 20 \log \left| \frac{0.75}{(5)} \right|$

$= -16.478 \approx -16.5$ db

$A_@ \underline{\omega = \omega_{c_2}} \Rightarrow A = 20 \log \left| \frac{0.75}{5} \right|$

$\left[\text{slope from } \omega_{c_1} \text{ to } \omega_{c_2} \times \log \frac{\omega_{c_2}}{\omega_{c_1}}\right] +$

$(0 * \log \frac{10}{5} + -16.478)$ = ~~-17.2498~~

$= -16.5$ db

$A_@ \underline{\omega = \omega_h} \Rightarrow A = 20 \log \left| \frac{0.75}{100} \right|$

$(\text{slope from } \omega_{c_2} \text{ to } \omega_h) \times \log \frac{\omega_h}{\omega_{c_2}}$

= ~~-17.2498~~

$-40 \times \log\left(\frac{100}{10}\right) + -16.5$

$= \boxed{-56.5 \text{ db}}$

we can't find the gain margin & phase margin with gain cross over frequency. Here all the gains are Negative Hence, there is no ω_{gc}. So, we select $\omega_l = 0.5$ rad/sec

$\omega_l = 0.5$ rad/sec $\longrightarrow$ $A = 3.5$ db.

Phase plot

ω rad/sec	$\tan^{-1}(0.2\omega)$ deg	$\tan^{-1}\left(\frac{0.16\omega}{1 - 0.1\omega^2}\right)$ deg	$\phi = \angle G(j\omega)$ deg
0.5	5.71	4.6	≈ -88
~~1~~	11.30	9.2	-88
5	45	46.8	-92
10	-63.43	90	-116
100	87.137	170.8	-174

$\phi_{gc} = 88°$

Phase margin $\gamma = 180° + \phi_{gc} = 180° - 88°$

$= 92°$

$\omega_{pc} = -180°$ cross changing point phase curve

Phase margin $\gamma = 92°$

phase plot crosses $-180°$ only at ∞

$|G(j\omega)| @ \infty$ is $-\infty$ db

Hence gain margin $= +\infty$

The phase plot crosses -180° only at ∞.
Hence, gain margein is + ∞.

Stability

1) A system is stable, if its o/p is bounded (finite) for any bounded (finite) i/p.

2) If an s/m o/p is stable for all variation of its parameters, then the s/m is called absolutely stable s/m.

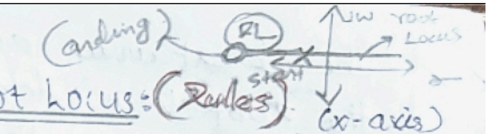
3) If a s/m o/p is stable for a limited range of variation of its parameters, then the s/m is called conditionally stable s/m.

4) A s/m is asymptotically stable, if in the absence of the i/p, the o/p tends towards zero irrespective of Intial condition

5) A s/m is unstable it for a bounded disturbing i/p signal the o/p is infinite amplitude or oscillatory.

6) A s/m is unstable if its o/p is unbounded of any of the bounded i/p.

Root LOCUS



Construction of Root Locus: (Rules)

Theory part imp

1) The Root Locus is symmetrical about real axis

2) Each branch of root Locus originate from an open loop pole and terminate at either on Finite open loop zero or open loop zero at ∞.

3) when we consider a point on real axis we can say that point is a part of root Locus or not: when the sum of poles and zeros on the right sides of that point is an odd number. then, the point is a part of Root Locus.

4) Asympteos is a line that passes to ∞ along with Root Locus.

ϕ Asymptacangle $\Rightarrow$ $\phi_A = \frac{180(2q+1)}{n-m}$

where, $q = 0, 1, 2, \ldots, n-m$
$n =$ no. of poles
$m =$ no. of zeros.

5) centroid is a point of intersection of the asymptone. It is on real axis.

centroid $\sigma_A = \frac{\text{sum of poles} - \text{sum of zero}}{n-m}$

6) The breakaway and breakin point of the root locus determined from the roots of the equation $dk/ds = 0$. If r numbers of branches of root locus meet at a point then, they break away at an angle of $\pm 180°/r$.

7) Angle of departure & angle of arrival :-

~~angle of departure~~

$$\boxed{\phi_p = \pm 180°(2q+1) + \phi}$$

$q = 0, 1, 2, \ldots (n-m)$

$$\boxed{\phi_z \Rightarrow \pm 180°(2q+1) + \phi \quad q = 0, 1, 2 \ldots}$$

(arrival)

8) point of intersection of root locus branches with the imaginary axis can be determined by use of the Routh criterion

9) The open loop gain k at any point $s = s_a$

$$k = \frac{\text{Product of vector length from open loop poles to the point } s_a}{\text{product of vector length from open loop zeros to the point } s_a}$$

Q) 3) A unity feedback control s/m has an open loop Transfer function, $G(s) = \frac{K}{s(s^2+4s+13)}$ sketch the root locus?

numerator il s-termila → ∴ no Zeros. $m = 0$.

1) To locate poles and zeros:

The poles of open loop T.F are the roots of the equation, $s(s^2+4s+13) = 0$.

No, no. of zeros ∴ $m = 0$

poles = root of denomenator

$= (s^2 + 4s + 13)s$

$(s+0) = 0$.

$P_1 = 0$.

$s = ?$ $\quad s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$s = -2 \pm j3$

$P_1 = 0$.

$P_2 = -2 + j3$

$P_3 = -2 - j3$

step-2

To find the Root Locus on the real axis

step-3

calculation angle of asymplones and centroid.

$$\text{angle of asymptoes} = \frac{\pm 180^\circ(2q+1)}{n-m} \quad q=0,1,2,..n-m$$

Here, $n=3$, and $m=0$.

$\therefore q = 0, 1, 2, 3$

$q=0$ Angle $= \frac{\pm 180^\circ}{3} = \pm 60^\circ$

$q=1$ Angle $= \pm \frac{180 \times \cancel{3}}{\cancel{3}} = \pm 180^\circ$

$q=2$ Angle $\frac{\pm 180 \times 5}{3} = \pm 300 = \mp 60^\circ$

$q=3$ Angle $= \frac{\pm 180 \times 7}{3} = \pm 420 = \pm 60^\circ$

To find centroid

$$\sigma_A = \frac{\text{sum of poles} - \text{sum of zeros}}{n-m}$$

$$= \frac{(0-2)+j3-2-j3-0}{3} = \frac{-4}{3}$$

$= -1.33$

step-4

To find the breakaway and breakin point

The closed loop Transfer function $\Big\} \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$ Here $H(s)=1$

$$= \frac{\frac{K}{s(s^2+4s+13)}}{1+\frac{k}{s(s^2+4s+13)}} \rightarrow s^3+4s+13$$

$$= \frac{K}{s(s^2+4s+13)+K}$$

The characteristic eqn is

$s(s^2+4s+13)+k=0.$

$s^3+4s^2+13s+k=0.$

$k = -s^3-4s^2-13s$

on differentiating the eqn of k with w.r.t; s we get

$$\frac{dk}{ds} = -(3s^2+8s+13)$$

Put $\frac{dk}{ds} = 0$

$-(3s^2+8s+13)=0 \Rightarrow$

$(3s^2+8s+13)=0$

$$s = \frac{-8 \pm \sqrt{8^2-4\times13\times3}}{2\times3}$$

$s = -1.33 \pm j1.6$

For getting the correct breakaway & break in point substitute the value of s in expression of K.

$K = -(s^3+4s^2+13s)$

$s = -1.33+1.6j$

$$K = -\left[(-1.33+1.6j)^3 + 4(-1.33+1.6j)^2 + 13\,(1.33+1.6j)\right]$$

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$(-1.33+1.6j)^3 = (-1.33)^3 + 3(-1.33)^2(1.6j) + 3(1.6)^2(-1.33) + (1.6j)^3$

$\Rightarrow -20.448$

$(-1.33+1.6j)^2 = a^2+b^2+2ab.$

$= (-1.33)^2 + (1.6j)^2 + 2\times(-1.33)(1.6j)$

$\Rightarrow$

$13\times(1.33+1.6j) \Rightarrow$

Here, the value of K is not real and positive. So, It is not consider as a breakaway / break in point

The break away / break in point is consider $s = -1.33 - 1.6j$

then, the K is.

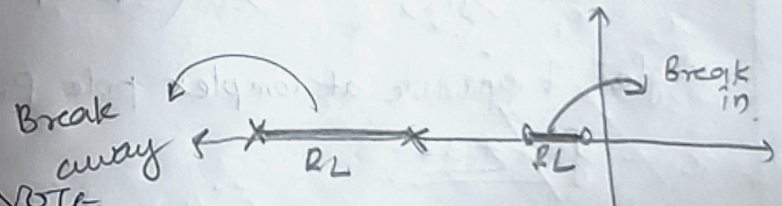
$$K = \left[(-1.33-1.6j)^3 + 4(-1.33-1.6j)^2 + 13\,(1.33-1.6j)\right]$$

$K = 12.592 + 8.17072j$

Here, the root locus that has neither break away nor breakin point.

Step-5

To find the angle of departure.



NOTE

1. If there is a root Locus on real axis b/w 2 poles then, there exist a break away point

3) If there is a root locus on real axis b/w 2 zeros then, there exist a break in point.

pole → departure
zero → arrival

(✱✱) step - 6

Angle of Departure (from a complex pole A) ⇒

$$180 - \begin{pmatrix}\text{Sum of angle of} \\ \text{vector to the complex pole} \\ \text{A from other poles}\end{pmatrix} + \begin{pmatrix}\text{sum of Angle} \\ \text{of vectors to the} \\ \text{complex pole} \\ \text{A from zeros}\end{pmatrix} \longrightarrow ①$$

Angle of arrival at a complex zero A, ⇒

$$180^\circ - \begin{pmatrix}\text{Sum of angle of vector} \\ \text{to the complex zero A} \\ \text{from all other zeros}\end{pmatrix} + \begin{pmatrix}\text{sum of angles of} \\ \text{vectors to the} \\ \text{complex zero} \\ \text{A from poles}\end{pmatrix}$$

Angle of departure at complex pole P_3

$$\underline{P_3 = +33}$$

To find the crossing point on imaginary axis:

The characteristic eqn, $s^3 + 4s^2 + 13s + k = 0$

put $s = j\omega$

$(j\omega)^3 + 4(j\omega)^2 + 13(j\omega) + k = 0$.

$\Rightarrow j\omega^3 - 4\cancel{j}\omega^2 + 13j\omega + k = 0$.

on equating imaginary part to zero, we get

$-\omega^3 + 13\omega = 0$.

$-\omega^3 = -13\omega$

$\omega^2 = 13$

$\omega = \pm\sqrt{13}$

$= \pm 3.6$

on equating real part to zero, we get

$-4\omega^2 + k = 0$

$k = 4\omega^2$

$= 4 \times 13$

$= \underline{52}$.

The crossing point of root locus is $\pm j3.6$. The value of k @ this crossing point $k = 52$.

Q) A s/m is charactrized by s. TF [Test controllabity & observability]

$$\frac{Y(S)}{U(S)} = \frac{2}{s^3+6s^2+11s+6}$$

Sol^n

$$Q_c = \left[B \quad AB \quad A^2B \cdots A^{n-1}B\right]$$
controllab

$$Q_o = \left[C^T \; A^TC^T \; (A^T)^2C^T \cdots (A^T)^{n-1}C^T\right]$$
observ

$$\frac{Y(S)}{U(S)} \times \frac{2}{s^3+6s^2+11s+6}$$

$$Y(S)\left[S^3+6S^2+11S+6\right] = 2U(S)$$

Taking ILT

$$\dddot{y}(t) + 6\ddot{y}(t) + 11\dot{y}(t) + 6y(t) = 2U(t)$$

$$x_1 = y(t)$$

$$x_2 = \dot{x}_1 = \dot{y}(t)$$

$$x_3 = \dot{x}_2 = \ddot{y}(t)$$

$$x_4 = \dot{x}_3 = \dddot{y}(t)$$

$$\dddot{y}(t) = 2u(t) - 6\ddot{y}(t) - 11\dot{y}(t) - 6y$$

$$\dot{x}_3 = 2u(t) - 6x_3 - 11x_2 - 6x_1$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \overset{A}{\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix}}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \overset{B}{\begin{bmatrix}0\\ 0\\ 2\end{bmatrix}}u$$

$$y = \underset{C}{\begin{bmatrix}1 & 0 & 0\end{bmatrix}}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$
(State model)

$n = 3$

$A^{n-1}B$

$$Q_c = \begin{bmatrix}B & AB & A^2B\end{bmatrix}$$

$$AB = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix}\begin{bmatrix}0\\ 0\\ 2\end{bmatrix} = \begin{bmatrix}0\\ 2\\ -12\end{bmatrix}$$

$$A^2B = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix}\begin{bmatrix}0\\ 2\\ -12\end{bmatrix} = \begin{bmatrix}2\\ -12\\ 50\end{bmatrix}$$

$$Q_c = \begin{bmatrix}0 & 0 & 2\\ 0 & 2 & -12\\ 2 & -12 & 50\end{bmatrix}$$

$$|Q_c| = 0\left(2\times 50 - (-12\times -12)\right) - 0\left(0\times 50 - 2\times -12\right)$$
$$+ 2\left((0\times -12) - (2\times 2)\right]$$
$$= 0 - 0 + 2(-4)$$
$$= -8 \Rightarrow \text{non-singular matrix}$$

rank $(3 = n)$ (controllable)

$$Q_o = \left[C^T \quad A^T C^T \quad (A^T)^2 C^T \right]$$

$y = [1\ 0\ 0]$

$$C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad A^T C^T = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$(A^T)^2 C^T = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Q_o = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|Q_o| = 1 \neq 0$$

rank = 3 = n

non - singular matrix

compeltely observable