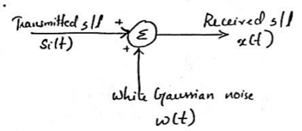
SIGNALLING OVER AWGN CHANNEL :

Signal Space Analysis :

→ Signal space analysis provides a mathematically elegant tool for the study of data transmission.

→



→ Upon the reception of $x(t)$ the receiver makes the best estimate of the message m_i

ie,

fig: Model for genetic digital communication system.

→ Best estimate means minimization of the avg. Probability of symbol error.

$$P_e = \sum_{i=1}^{M} P_i \cdot \hat{P}(\hat{m} \neq m_i / m_i) \longrightarrow (1)$$

→ Based on this criterion we can begin to design the receiver that can give the best decision

→ A message source emits one symbol every T secs, with the symbols belonging to an alphabet of M symbols.

→ The txr takes the message source o/p & codes it into distinct s/l $s_i(t)$ suitable for transmission over the channel.

→ $s_i(t)$ occupies full duration T alloted to symbol & $s_i(t)$ is a real valued energy signal (ie s/l with finite energy).

ie, $E_i = \int_0^T s_i^2(t)\, dt$, $i = 1, 2, \dots M$

→ The channel should be :

1) Linear with a BW wide enough to accomodate the txn of $s_i(t)$ with no distortion.

2) Channel noise $w(t)$ is the sample function of zero-mean white Gaussian noise process.

ie AWGN (Additive White Gaussian Noise)

(3)

→ $x(t) = s_i(t) + w(t) \quad \begin{cases} 0 \le t \le T \\ i = 1, 2, \dots M \end{cases}$

→ Owing to the presence of channel noise, the decision making process is statistical & the rxr will make occasional errors.

→ As / eqn (1) the requirement is to design the receiver to minimize the avg. probability of error. which is satisfied in eqn (1)

where P_i = priori probability

$P(\hat{m} \neq m_i / m_i)$ = conditional probability

Gram Schmidt Orthogonalization procedure:

Gram schmidt orthogonalization procedure provides a complete orthonormal set of basis functions.

→ Suppose we have a set of M energy s/ls denoted by $s_1(t), s_2(t) \dots s_M(t)$.

→ Starting with $s_1(t)$, first basis function is defined by,

$$\boxed{\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}}$$

where E_1 = energy of s/l $s_1(t)$.

$$s_1(t) = \sqrt{E_1}\,\phi_1(t)$$
$$= s_{11}\,\phi_1(t)$$

→ Next using $s_2(t)$, s_{21} is defined

$$s_{21} = \int_0^T s_2(t)\,\phi_1(t)\,dt$$ ie projection of $s_2(t)$ onto the basis of $\phi_1(t)$

→ New intermediate function, $g_2(t)$ introduced,

$$g_2(t) = s_2(t) - s_{21}\phi_1(t)$$

$g_2(t)$ is orthogonal to $\phi_1(t)$.

$$\boxed{\phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)\,dt}} = \frac{s_2(t) - s_{21}\phi_1(t)}{\sqrt{E_2 - s_{21}^2}}}$$

Generalizing $g_i(t)$ is,

→ $$g_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij}\,\phi_j(t)$$

→ where, $s_{ij} = \int_0^T s_i(t)\,\phi_j(t)\,dt$, $j = 1, 2, \dots i-1$

→ Basis function are,

$$\phi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)\,dt}}, \quad i = 1, 2, \dots N$$

→ The dimension N is less than or equal to the number of given signals, M, depending on whether the s/ls are linearly independent or not.

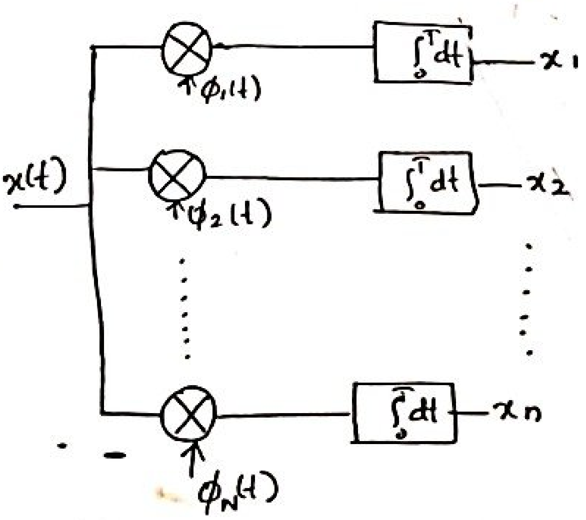
where gauss... de...

1

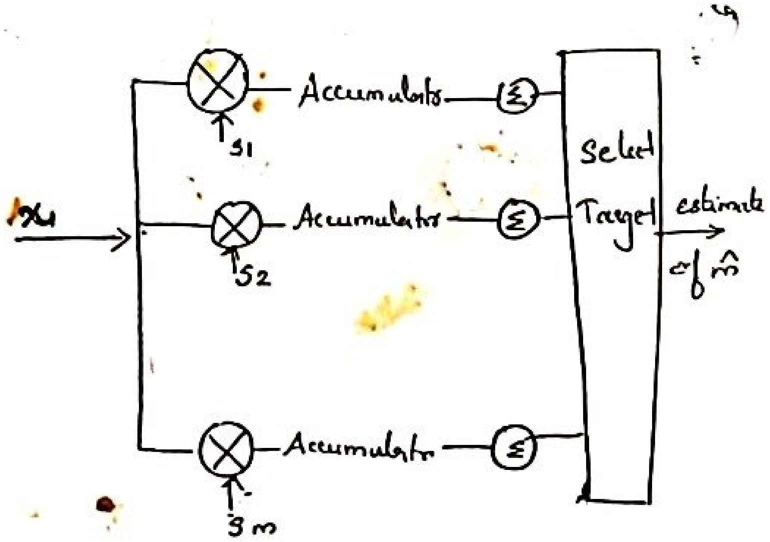
Correlation receiver :

→ Correlation receiver consists of 2 parts - detector & decoder. Detector consists of m product integrators or correlators supplied with a corresponding set of orthonormal basis functions $\phi_1(t), \phi_2(t) \ldots \phi_N(t)$.

→ These correlators operate on the rxd s/l $x(t)$ to produce observation vector x.



Detector



Decoder

→ Second part is the s/l txn decoder.

→ It is implemented in the form of maximum likelihood decoder that operates on observation vector x to produce an estimate of $\hat{m}$ of the txtd s/l. m_i.

→ In such a way to minimize the avg. probability of the s/l error, 'n' elements of observation vector x are multiplied by 'm' s/l vector & the products

products are summed in accumulator to produce the set of inner products.

→ A logic control will select the largest of the s/l & produce the estimate $\hat{m}$.

Probability of error

→ Assume that observation space 'z' is partitioned in accordance with maximum likelihood decision rule into a set of 'M' regions. If a symbol 'm' transmitted then an observation vector 'x' is received. An error occurs when the received s/l represented by x doesnot fall inside the region z.

→ Avg. probability error,

$$P_e = \sum_{i=1}^{M} P[x \text{ doesnot lie in } z / m \text{ sent}]$$

$$= \frac{1}{M} \sum_{i=1}^{M} P[x \text{ doesnot lie in } z / m \text{ sent}]$$

$$= 1 - \frac{1}{M} \sum_{i=1}^{M} P[x \text{ lies in } z / m \text{ sent}]$$

MATCHED FILTER:

→ It is an optimum filter used to detect input signa[l] from the Additive White Gaussian Noise. The power spectral density is, $S_{ni}(f) = \frac{N_0}{2}$ → (1)

Impulse response for the Matched filter:

→ Transfer function of optimum filter is,

$$H(f) = k \frac{X^*(f)}{S_{ni}(f)} e^{-j2\pi fT} \rightarrow (2)$$

subst. (1) in (2),

$$H(f) = k \frac{X^*(f)}{N_0/2} \cdot e^{-j2\pi fT}$$

$$H(f) = \frac{2k}{N_0} X^*(f) e^{-j2\pi fT} \rightarrow (3)$$

from the prop. of Fourier Transform,

$$X^*(f) = X(-f) \rightarrow (4)$$

subst (4) in (3),

$$H(f) = \frac{2k}{N_0} X(-f) e^{-j2\pi fT} \rightarrow (5)$$

Impulse response of matched filter is evaluated by t[he] inverse fourier transform of eqn (5),

$$h(t) = IFT[H(f)] = IFT\left[\frac{2k}{N_0} X(-f) e^{-j2\pi fT}\right] \rightarrow (6)$$

$$FT \text{ of } x(T-t) = X(-f) e^{-j2\pi fT} \rightarrow (7)$$

Subst (7) in (6),

$$h(t) = \frac{2k}{N_0} x(T-t)$$

Let $x(t) = x_1(t) - x_2(t)$,

$h(t) = \frac{2k}{N_0}[x_1(T-t) - x_2(T-t)]$ gives the required impulse response of the matched filter.

Probability of Error (Pe) for matched filter or Error rate to noise:

Error probability of optimum filter is,

$$Pe = \frac{1}{2} \operatorname{erfc}\left[\frac{x_{01}(T) - x_{02}(T)}{2\sqrt{2}\,\sigma}\right] \rightarrow (1)$$

In the above eqn,

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \int_{-\infty}^{\infty} \frac{|X(f)|^2}{S_{ni}(f)}\, df$$

$$= \int_{-\infty}^{\infty} \frac{|X(f)|^2}{\frac{N_o}{2}}\, df = \frac{2}{N_o}\int_{-\infty}^{\infty} |X(f$$

$\rightarrow (2)$

& As/ Parsevals power theorem,

$$\int_{-\infty}^{\infty} |X(f)|^2 df = \int_{-\infty}^{\infty} x^2(t)\, dt = \int_0^T x^2(t)\, dt \rightarrow (3)$$

Eqn (2) can be written as/ eqn (3).

$$\int_{-\infty}^{\infty} |X(f)|^2 df = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$= \int_0^T [x_1(t) - x_2(t)]^2 dt$$

$$= \int_0^T [x_1^2(t) + x_2^2(t) - 2x_1(t)x_2(t)]\, dt$$

(5)

the

$$\int_0^T x_1^2(t)\, dt = E_1 \quad \text{ie, energy of } x_1(t)$$

$$\int_0^T x_2^2(t)\, dt = E_2 \quad \text{ie, energy of } x_2(t)$$

& $\int_0^T x_1(t)x_2(t) = E_{12}$ ie, energy due to autocorrelation between $x_1(t)$ & $x_2(t)$.-

if $x_1(t) = -x_2(t)$ then, $E_1 = E_2 = -E_{12} = E$

$$\text{ie, } \int_{-\infty}^{\infty} |X(f)^2|\, df = E + E - 2(-E) = 4E$$

&,

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \frac{2 \cdot 4E}{N_o} = \frac{8E}{N_o}$$

therefore,

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]_{max} = 2\sqrt{2}\sqrt{\frac{E}{N_o}} \rightarrow (a)$$

Subst eqn (a) in Pe,

$$\boxed{Pe = \frac{\quad}{2} \ldots \sqrt{\;\; N_o}}$$

$(SNR)_o$ of matched filter :

→ SNR of optimum filter is,

$$\left(\frac{S}{N}\right)_{o\,max} = \int_{-\infty}^{\infty} \frac{|X(f)|^2}{S_{ni}(f)}\, df \rightarrow ①$$

$$= \frac{2}{N_0} \int_{-\infty}^{\infty} |X(f)|^2 df \rightarrow ②$$

Rayleighs energy theorem states that,

$$\int_{-\infty}^{\infty} |X(f)|^2 df = \int_{-\infty}^{\infty} x^2(t)\, dt = E \rightarrow ③$$

from ③, eqn ② becomes,

$$\left(\frac{S}{N}\right)_{o\,max} = \frac{2}{N_0} E$$

Hence, max. signal to noise power of matched filter will

$$\left(\frac{S}{N}\right)_{o\,max} = \frac{2E}{N_0}$$

Now this eqn can be rearranged as under,

$$\left(\frac{S}{N}\right)_{o\,max} = \frac{E}{\frac{N_0}{2}}$$

Here, E is energy of s/l $x(t)$ & $\frac{N_0}{2}$ is power spectral density of white noise. Thus, we have :

$$\left(\frac{S}{N}\right)_{o\,max} = \frac{\text{Energy of the signal } x(t)}{\text{Psd of white noise}} \Rightarrow \text{Hence proved}$$

NYQUIST CRITERION FOR DISTORTIONLESS BASEBAND BINARY TRANSMISSION:

→ In the absence of ISI,

$$y(t_i) = \mu a_i$$

→ In order to minimize effects of ISI, the transmitting & receiving filters are designed properly

→ The transfer function of the channel & the shape of transmitted pulse generally specified.

→ from this the transfer functions of transmitting & receiving filters to reconstruct the transmitted data sequence $\{b_k\}$ is determined.

→ It is achieved by first extracting & then decoding the corresponding sequence of weights from o/p $y(t)$.

→ As expressed by following equation.

$$y(t) = \mu \sum_{k=-\infty}^{\infty} a_k \, p(t - kT_b).$$

This shows o/p $y(t)$ is dependent on a_k

Extraction:

→ It is basically the process of sampling. The s/l $y(t)$ is sampled at $t = iT_b$.

Decoding:

Decoding should be such that the contribution of weighted pulse i.e., $a_k \, p(iT_b - kT_b)$ for $i = k$ be free from ISI. This can be stated as,

$$p(iT_b - kT_b) = \begin{cases} 1 & \text{for } i = k \\ 0 & \text{for } i \neq k \end{cases}$$

where $p(0) = 1$ due to normalizing. → ①

5.26. RAISED COSINE SPECTRUM

1. Basic concept and Mathematical Expression

The two difficulties experienced by the ideal Nyquist channel can be overcome by increasing the bandwidth from its minimum value $B_0 = R_b/2$ to an adjustable value between B_0 and $2\,B_0$. A condition is put on the overall frequency response P(f) to satisfy the given condition.

As per equation (5.50), we have

$$\sum_{n=-\infty}^{\infty} P(f - nR_b) = T_b = \frac{1}{R_b}$$

$P(f) = \frac{1}{2B0}\,\text{rect}$

Expanding the summation sign, we get

$$\ldots P(f + R_b) + P(f) + P(f - R_b) + P(f - 2R_b) \ldots = T_b$$

$P(f) = \frac{1}{2B0}\,\text{rect}\,\frac{f}{2B0}$

But $\qquad B_0 = \frac{R_b}{2}, \quad$ therefore, $\quad R_b = 2B_0$

Thus, we write

$$\ldots P(f + 2B_0) + P(f) + P(f - 2B_0) + P(f - 4B_0) + \ldots = \frac{1}{2B_0}$$

We retain only the three terms on LHS which correspond to $n = -1$, $n = 0$ and $n = 1$ and restrict the frequency band of interest to $(-B_0, B_0)$ to get,

$$P(f + 2B_0) + P(f) + P(f - 2B_0) = \frac{1}{2B_0} \text{ and given } -B_0 \le f \le B_0$$

It is possible to derive several bandlimited functions which will satisfy above equation, one of them is called as the raised cosine spectrum. This spectrum consists of a flat portion and a roll off portion. The raised cosine spectrum is expressed mathematically as under:

$$P(f) = \begin{cases} 1/2B_0 & \text{(flat portion)} \quad \ldots 0 \le |f| < f_1 \\ \dfrac{1}{4B_0}\left\{1 - \sin\left[\dfrac{\pi(|f| - B_0)}{2B_0 - 2f_1}\right]\right\} & \ldots .\ f_1 \le |f| < 2B_0 - f_1 \\ 0 & \ldots\ |f| \ge 2B_0 - f_1 \end{cases}$$

The relation between frequency parameter f_1 and the bandwidth B_0 are related as under :

$$\alpha = 1 - \frac{f_1}{f_m}$$

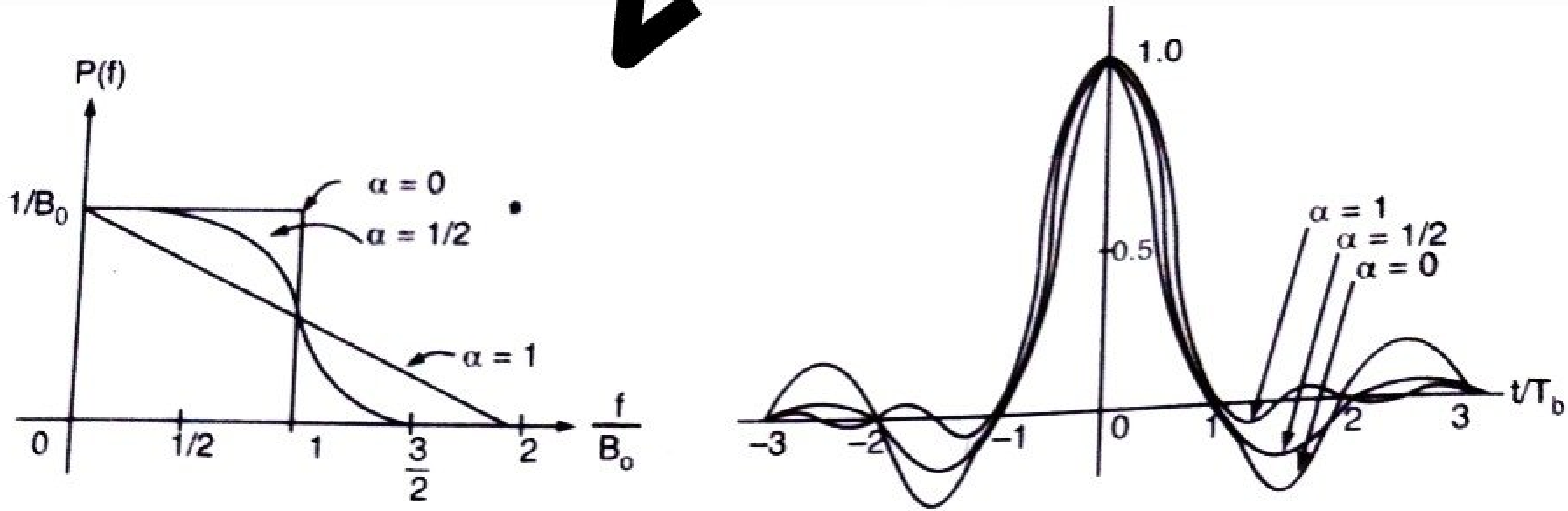
where α is called as the roll off factor. It indicates the excess bandwidth over the ideal solution B_0.

The transmission bandwidth B_T is defined as under :

$$B_T = 2B_0 - f_1 = B_0(1 + \alpha)$$

The normalized frequency response of raised cosine function is obtained by multiplying P(f) by $2B_0$ and it is plotted in figure 5.37(a), for different values of α. The corresponding time response p(t) is shown in figure 5.37(b).

2



(a) Frequency response

(b) Time response

Fig. 5.37. Responses for different roll-off factors, α

5.24 NYQUIST'S CRITERION FOR DISTORTIONLESS BASEBAND BINARY TRANSMISSION

1. Basic concept

In the previous section, we have observed that in absence of the ISI, we have

$$y(t_i) = \mu a_i \qquad ...(5.4$$

This expression shows that under these conditions, the i^{th} transmitted bit can be decode correctly. In order to minimize the effects of ISI, we have to design the transmitting and receivir filters properly. The transfer function of the channel and the shape of transmitted pulse ar generally specified. Therefore, it becomes the first step towards design of filters. From this informatic we have to determine the transfer functions of the transmitting and receiving filters, to reconstru the transmitted data sequence $\{b_k\}$. This is achieved by first extracting and then decoding th corresponding sequence of weights from the output y(t).

As expressed by following equation

$$y(t) = \mu \sum_{k=-\infty}^{\infty} a_k\, p(t - kT_b) \qquad ...(5.4$$

This shows that output y(t) is dependent on a_k, the received pulse p(t) and the scaling factor

2. Extraction

Extraction is basically the process of sampling. The signal y(t) is sampled at $t = iT_b$.

3. Decoding

The decoding should be such that the contribution of the weighted pulse i.e., $a_k\, p(iT_b - kT$ or $i = k$ be free from ISI. This can be stated mathematically as under:

$$p(iT_b - kT_b) = \begin{cases} 1 & \text{for } i = k \\ 0 & \text{for } i \neq k \end{cases} \qquad ...(5.4$$

where p(0) = 1 due to normalizing.

If p(t) i.e., received pulse satisfies the above expression, then the receiver output given by equation (5.45) reduces to

$$y(t_i) = \mu a_i \qquad ...(5.47)$$

which indicates zero ISI in the absence of noise.

5.27.2. Duobinary Signaling

The basic duobinary signaling scheme has been shown in figure 5.39

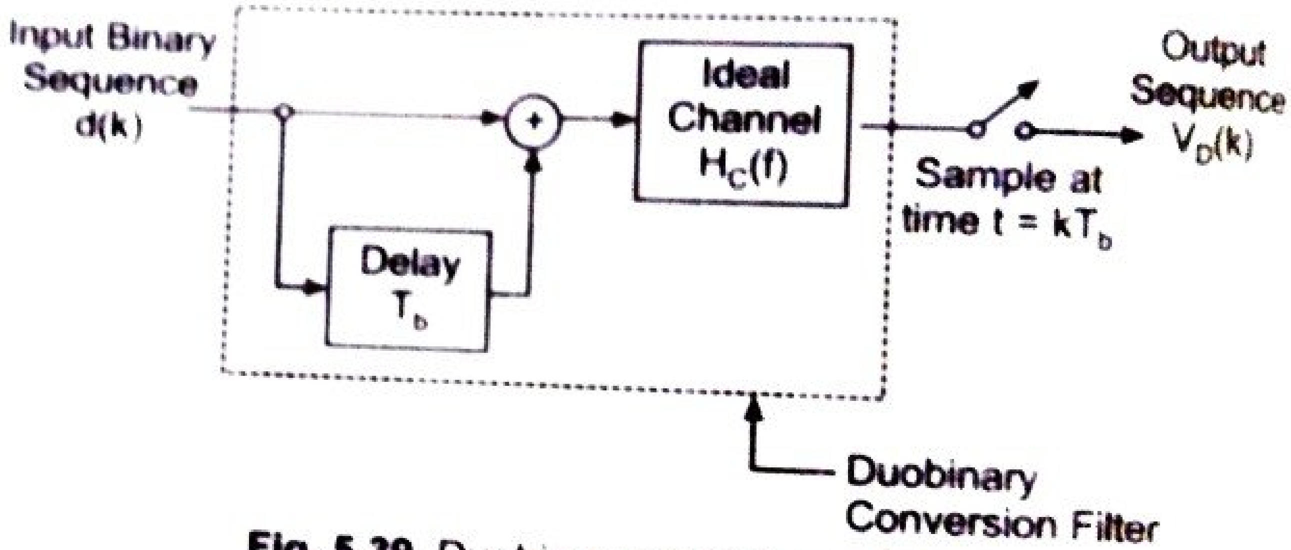


Fig. 5.39. Duobinary signaling scheme

The binary signal d(k) is first passed through a simple filter which consists of a single delay element. The present signal d(k) and its delayed version d(k − 1) are then added to get the duobinary signal at the output of the coder. This signal is mathematically expressed as:

$$V_D(k) = d(k) + d(k-1) \quad \ldots(5.54)$$

One of the important effects of this transformation is that the sequence {d(k)} of the uncorrelated binary digits at the input is converted into a sequence $V_D(k)$ of correlated digits. This correlation between the adjacent transmitted pulses is equivalent to introducing intersymbol interference (ISI) in the transmitted signal. However this ISI is under designers control and this is the basis of the correlative coding.

Maximum Likelihood Detection

If the observation vector x is given then we can perform a mapping from x to get an estimate $\hat{m}_i$ of a transmi-tted signal m_i. To minimize the avg. probability of symbol error in decision

$$P_e(m_i, x) = P(m_i \text{ not sent} / x) = P(m_i \text{ sent} / x)$$

5.31.2. Tapped Delay Line Filter

Let us consider a time-invariant filter with an impulse response h(t). We assume that, h(t) = 0 for t < 0, i.e., the filter is causal. The impulse response of the filter is of finite duration i.e. h(t) = 0 for $t \geq T_f$.

(i) We can express the filter output y(t) produced in response of input x(t) as under:

$$y(t) = \int_0^{T_f} h(t).x(t - \tau)\, d\tau \qquad ...(5.93)$$

(ii) Let the input x(t), impulse response h(t) and output h(t) and output y(t) be uniformly sampled at a rate of $1/\Delta\tau$ samples per second.

therefore $\quad t = n\Delta\tau \qquad (5.94)$

and $\quad \tau = k\,\Delta\tau \qquad ...(5.95)$

where, n and k both are integers and $\Delta\tau$ is the sampling period.

(iii) If the value of $\Delta\tau$ is very small, then, the product $h(t)\,x(t - \tau)$ of equation (5.93) will remain constant for $k\,\Delta\tau \leq \tau \leq (k + 1)\Delta\tau$ for all values of k and t. Then, the equation (5.93) may be approximated by the convolution sum as under :

$$y(n\,\Delta\,\tau) = \sum_{k=0}^{N-1} h(k\,\Delta\tau).\,x(n\Delta\tau - k\,\Delta\,\tau)\,\Delta\tau \qquad ...(5.96)$$

when, $N\,\Delta\tau = T_f$

Substituting $h(k\,\Delta\tau)\,.\,\Delta\tau = w_k$ into equation (5.96), we get,

$$y(n\,\Delta T) = \sum_{k=0}^{N-1} w_k\,.\,x(n\,\Delta\tau - k\,\Delta\tau) \qquad ...(5.97)$$

(iv) Equation (5.97) can be realized using the circuit shown in figure 5.68 which is known as a tapped delay line filter or transversal filter. Because, if we expand equation (5.97), then, we get,

$$y(n\,\Delta\tau) = w_0 x(n\,\Delta\tau) + w_1\,x\,(n\,\Delta\tau - \Delta\tau) + w_2\,x(n\,\Delta\tau - 2\,\Delta\tau) + ... + w_{N-1}\,x\,[n\,\Delta\tau - (N-1)\,\Delta\tau] \qquad ...(5.98)$$

This expression is realized as shown in figure 5.69.

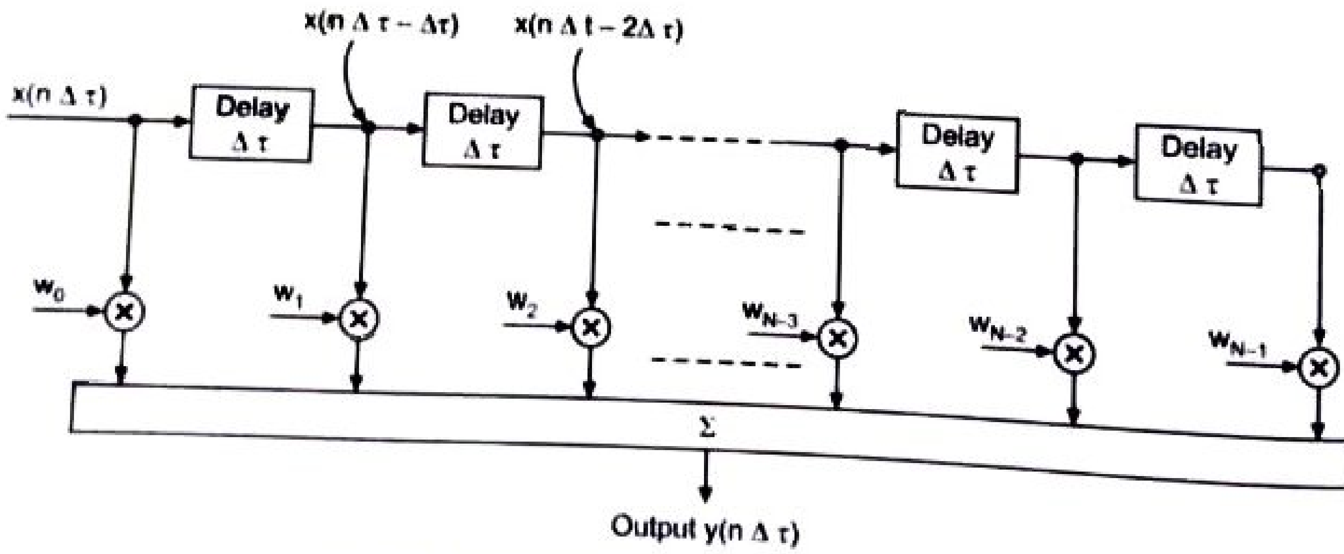


Fig. 5.69. Tapped delay line filter or transversal filter

5.31.3 Automatic Equalizers

In order to solve a set of simultaneous equations, the tap gains of the zero forcing equalizer described in the previous section are adjusted. This is called as the **trimming** of the equalizers. The process of equalizer trimming involves the following steps.

Trimming Procedure

(i) First, we send a pulse through the system

[redacted]

(ii) Then, we measure the output at the receiving filter output at the proper sampling instants.

(iii) Next, we calculate the values of the tap gains.

(iv) Accordingly, we set the gains on the taps of the equalizer.

In the automatic equalizers, the tap gains are adjusted automatically with the help of accurate and simple automatic system. These automatic systems are classified into two types.

(a) Preset type equalizers

(b) Adaptive equalizers

The preset type equalizers used a special sequence of pulses before to or during the breaks in the data transmission. The adaptive type equalizers adjusts itself continuously during the transmission of data by operating on the data signal itself.

The automatic equalizers adjust the tap gains precisely at the optimum values by using the iterative technique.

5.31.4. Preset Equalizer

The block diagram of the preset equalizer has been shown in figure 5.70. Here, the components of the error vector are measured by transmitting a sequence of widely separated pulses through the system.

The output of the equalizer is measured at the proper instants of time. The tap gains are adjusted with the help of fixed iterations of the step size Δ. The sampling of the filter output at proper instants is done by using a timing circuit. This timing circuit is triggered by the peak detector. The center sample is compared with + 1 (or sliced at + 1) and the polarity of the error component ε_0^k is obtained. The polarities of the remaining error components are obtained by using the value of the filter output at $t = \pm jT_s$. The gate is opened at the end of the k^{th} test pulse. Depending on the polarity of the error component ε^k the tap gains are increased or decreased by Δ accordingly. This procedure is known as the training procedure. This procedure can take hundreds of pulses.

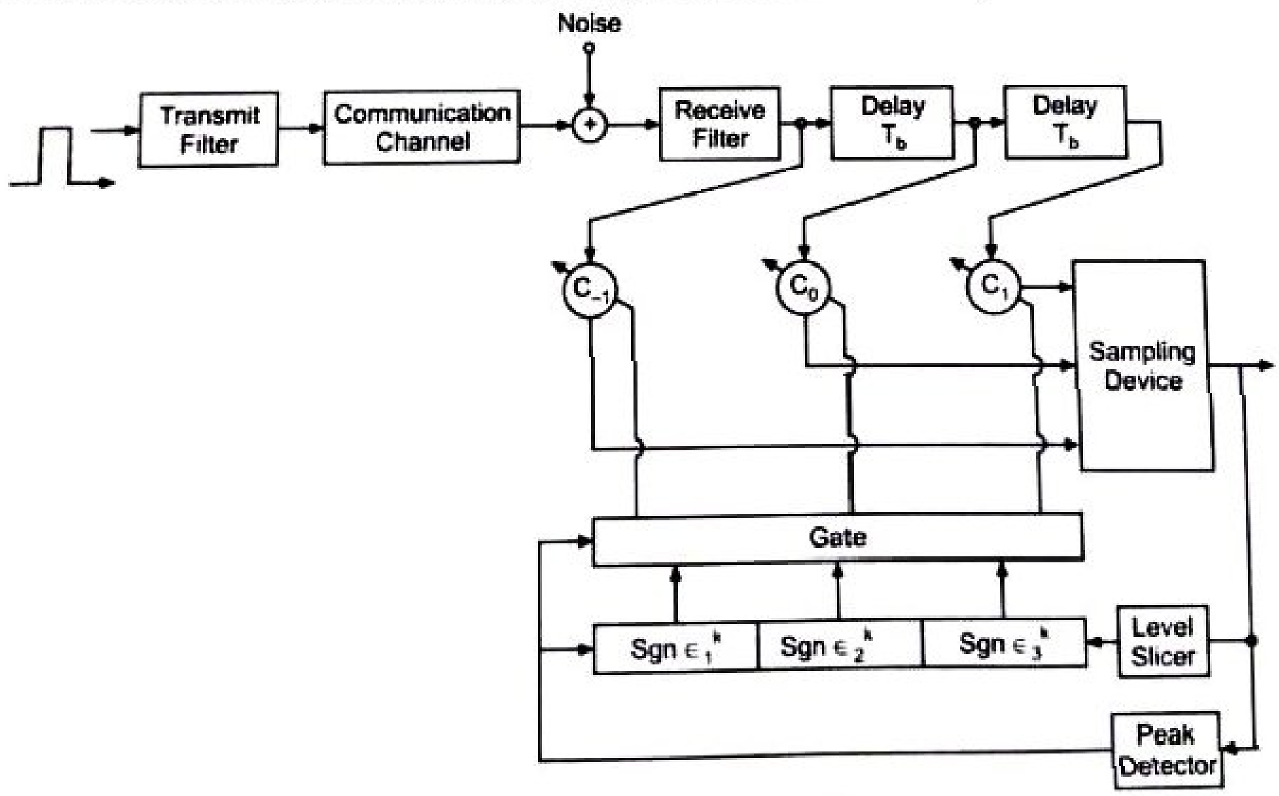


Fig. 5.70. A three tap preset equalizer

.31.5 **Adaptive Equalizer**

The block diagram of an adaptive equalizer has been shown in figure 5.71. In the three tap

adaptive equalizer of figure 5.71, the error vector ε^k is estimated continuously when the normal data transmission is going on. This is called as the adaptive equalizer because this scheme has the ability to change during the data transmission which eliminates the need for long training sessions, which is, therefore, the preset equalizer. The advantages of the adaptive equalizers are that they are more versatile, more accurate and cheaper than the preset equalizers.

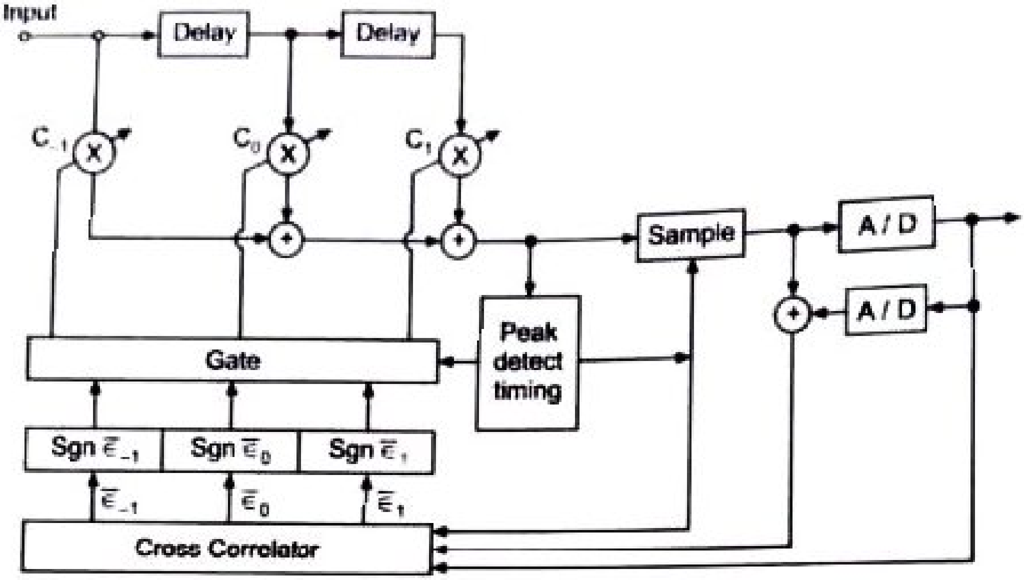


Fig. 5.71. A three tap adaptive equalizer

5.31.6 Decision Feedback Equalizer (DFE)

The basic limitation of a linear equalizer, such as the transversal filter, is the poor perform on channel having spectral nulls. A decision feedback equalizer (DFE) is a non-linear equalizer that uses previous detector decision to eliminate the ISI on pulses that are currently being demodulated. In other words, the distortion on a current pulse that was caused by previous pulses is subtracted. Figure 5.72 shows a simplified block diagram of a DFE where the forward filter and the feedback filter can each be a linear filter, such as transversal filter.

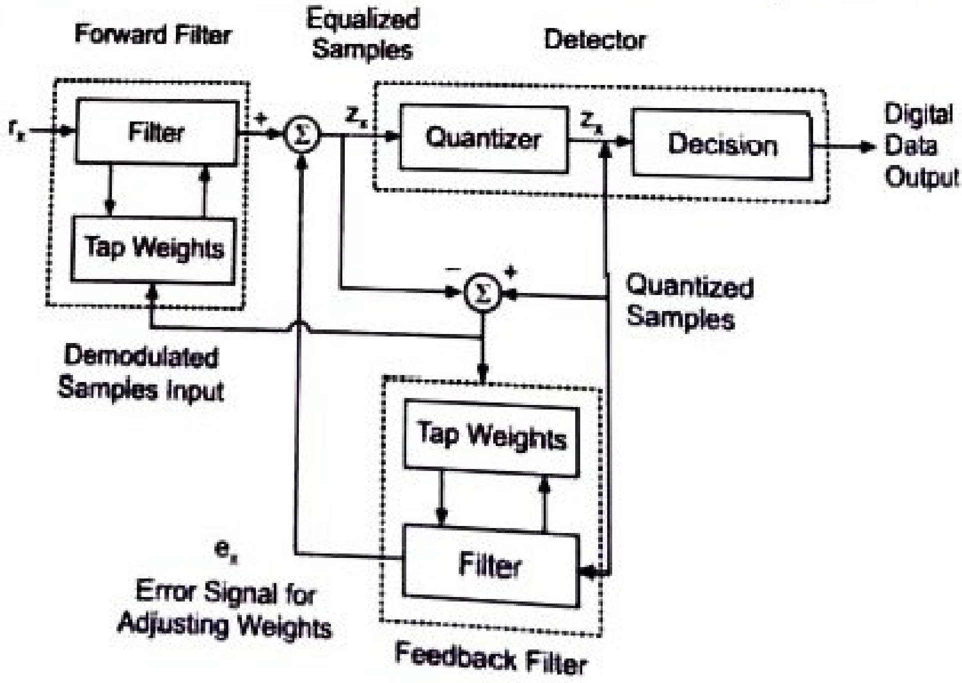


Fig. 5.72.

The non-linearity of the DFE stems from the non-linear characteristics of the detector that provides an input to be feedback filter. The basic idea of a DFE is that if the values of the symbols previously detected are known, then ISI contributed by these symbols can be cancelled out exactly at the output of the orward filter by subtracting past symbol values with appropriate weighting. The forward and feedback ıp weights can be adjusted simultaneously to fulfill a criterion such as minimizing the MSE. The lvantage of a DFE implementation is the feedback filter, which is additionally working to remove I, operates on noiseless quantized levels, and thus its output is free from channel noise.

Maximum a posteriori (MAP)

Fitting

As the name suggests we find the parameters which maximize the posterior probability $Pr(\boldsymbol{\theta}|\mathbf{x}_{1\ldots I})$.

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmax}}\left[\frac{\prod_{i=1}^{I} Pr(\mathbf{x}_i|\boldsymbol{\theta})Pr(\boldsymbol{\theta})}{Pr(\mathbf{x}_{1\ldots I})}\right]$$

Since the denominator doesn't depend on the parameters we can instead maximize

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmax}}\left[\prod_{i=1}^{I} Pr(\mathbf{x}_i|\boldsymbol{\theta})Pr(\boldsymbol{\theta})\right]$$

GENERALISED PARTIAL RESPONSE SIGNALLING: OR CORRELATIVE CODING:

→ The duobinary & modified duobinary techniques have correlation spans of 1 binary digit & 2 binary digits respectively.

→ These 2 techniques are generalized to other schemes, which are known collectively as correlative coding schemes.

→ It is shown below, where $H_c(f)$ is defined earlier i.e., $H_c(f) = \begin{cases} 1, & |f| \le R_b/2 \\ 0, & \text{otherwise} \end{cases}$

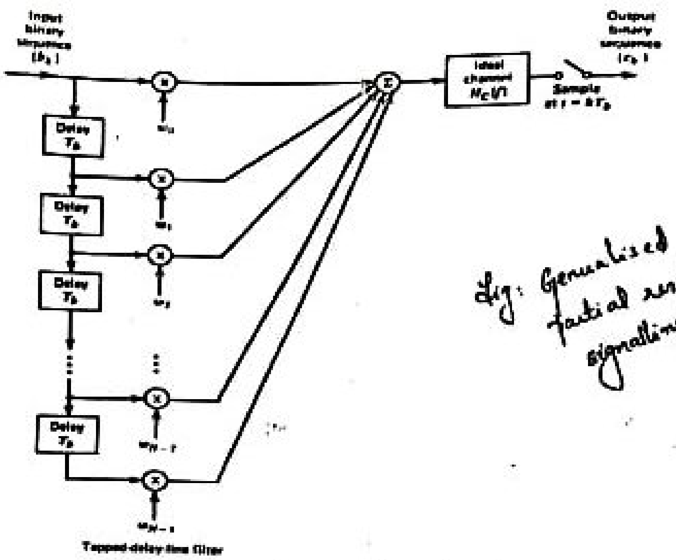


fig: Generalised partial response signalling.

→ It involves use of tapped delay-line filter with tap weights $w_0, w_1, \dots, w_{N-1}$. A correlative sample c_k is obtained from superposition of N successive i/p sample values b_k as,

$$c_k = \sum_{n=0}^{N-1} w_k b_{k-n} \rightarrow (1).$$

→ By choosing various combinations of integer values for w_n different forms of correlative coding schemes to suit individual applications are obtained.

→ for duobinary we have, $w_0 = +1$, $w_1 = +1$ & $w_n = 0$ for $n \ge 2$

→ In modified duobinary we have, $w_0 = +1$, $w_1 = 0$, $w_2 = -1$ & $w_n = 0$: $n \ge$

$$c_k = \sum_{n=0}^{N-1} w_k b_{k-n}. \quad w..$$

8.14 CAPACITY OF AN ADDITIVE WHITE GAUSSIAN NOISE (AWGN) CHANNEL SHANNON-HARTLEY LAW

In an additive white Gaussian noise (AWGN) channel, the channel output Y is given by

$$Y = X + n$$

where X is the channel input and n is an additive bandlimited white Gaussian noise with zero mean and variance σ^2.

The capacity C_s of an AWGN channel is given by

$$C_s = \max_{\{f_X(x)\}} I(X;Y) = \frac{1}{2}\log_2\left(1+\frac{S}{N}\right) \text{ b/sample} \quad \text{...(8.49)}$$

where S/N is the signal-to-noise ratio at the channel output. If the channel bandwidth B Hz is fixed, then the output y(t) is also a bandlimited signal completely characterized by its periodic sample values taken at the Nyquist rate 2B samples/s.

Then the capacity C(b/s) of the AWGN channel is given by

An optimist is one who makes the best of it when the get...

$$C = 2B \times C_s = B\log_2\left(1+\frac{S}{N}\right) \text{ b/s} \quad \text{...(8.50)}$$

Equation (8.50) is known as the Shannon-Hartley law.

Important Point: The Shannon-Hartley law underscores the fundamental role of bandwidth and signal-to-noise ratio in communication. It also shows that we can exchange increased bandwidth for decreased signal power for a system with given capacity C.

Inter Symbol Interference (ISI):

→ when data is being transmitted in the form of pulses (ie bits) the o/p produced at the rxr due to other bits or symbols interfere with the o/p produced by the desired bit. This is Intersymbol Interference (ISI)

→ ISI introduces errors in detected s/l.

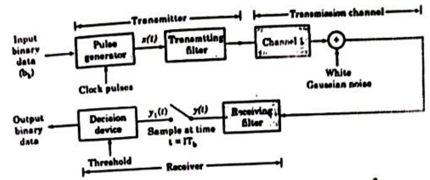


fig: Baseband binary data txn s/m

→ The fig above shows element of binary PAM s/m. The i/p s/l consists of binary data sequence $\{b_k\}$ with bit duration T_b seconds.

→ This sequence is applied to pulse generator to produce discrete PAM s/l which given by,

$$x(t) = \sum_{k=-\infty}^{\infty} a_k v(t-kT_b)$$

where $v(t)$: basic pulse ; when normalized $v(0) = 1$

→ Pulse amplitude modulator converts this i/p sequence into polar form as. if, $b_k = 1$ then $a_k = 1$
$b_k = 0$ then $a_k = -1$

a) The PAM s/l $x(t)$ is passed through transmitting filter. The o/p of which is txtd & over transmission channel. The impulse response of this channel = $h(t)$.

b) A random noise is then added to the txd factor s/l when it travels over the transmission channel. Thus the s/l rxd at the receiving end is contaminated with noise.

c) Channel o/p is passed through a receiving filter. This filter o/p is sampled synchronously with the txr. Sampling instants are determined by clock or timing s/l.

d) Sequence of samples obtained at the o/p of rxng filter is used to reconstruct the original data sequence.

e) Each sample is compared to a threshold level in the decision making device. If the amplitude of the sample is higher than the threshold level then it is decided that symbol 1 is rxd. On the other hand if the signal has an amplitude lower than the threshold then '0' is rxd.

The rxng filter o/p is,

$$y(t) = \mu \sum_{k=-\infty}^{\infty} a_k p(t-kT_b) + n(t)$$

μ = scaling factor
$n(t)$ = noise at o/p of rxng filter due to added noise
$p(t-kT_b)$ = combined impulse response of rxng filter.

→ Rxng filter o/p $y(t)$ is sampled at time $t_i = iT_b$

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} a_k \, p(iT_b - kT_b) + n(t_i)$$