Binary Phase Shift Keying

bpsk

The most straightforward type of PSK is called binary phase shift keying (BPSK), where "binary" refers to the use of two phase offsets (one for logic high, one for logic low).

We can intuitively recognize that the system will be more robust if there is greater separation between these two phases—of course it would be difficult for a receiver to distinguish between a symbol with a phase offset of 90° and a symbol with a phase offset of 91°. We only have 360° of phase to work with, so the maximum difference between the logic-high and logic-low phases is 180°. But we know that shifting a sinusoid by 180° is the same as inverting it; thus, we can think of BPSK as simply inverting the carrier in response to one logic state and leaving it alone in response to the other logic state.

To take this a step further, we know that multiplying a sinusoid by negative one is the same as inverting it. This leads to the possibility of implementing BPSK using the following basic hardware configuration:

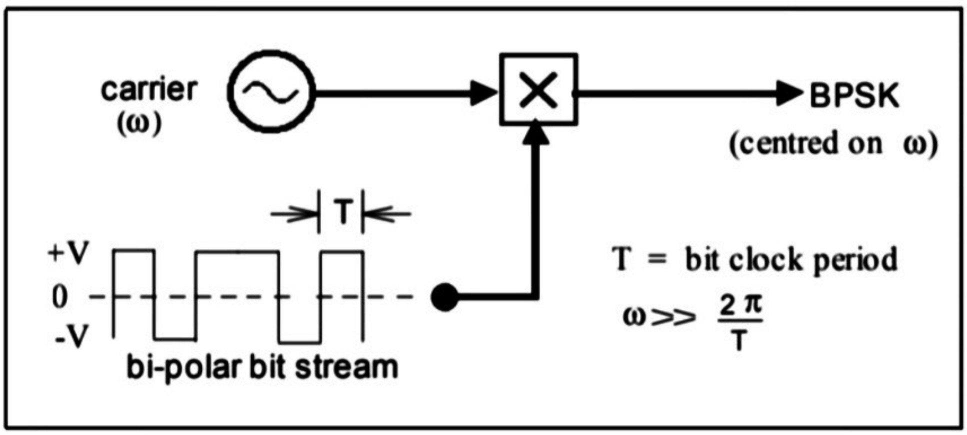


Figure 1: generation of BPSK

bpsk constellation

This signal space diagram is shown in Figure. If the symbols are equiprobable then the rule for deciding which symbol was transmitted is to choose the closest message point. The BPSK transmission system is illustrated as Figure.

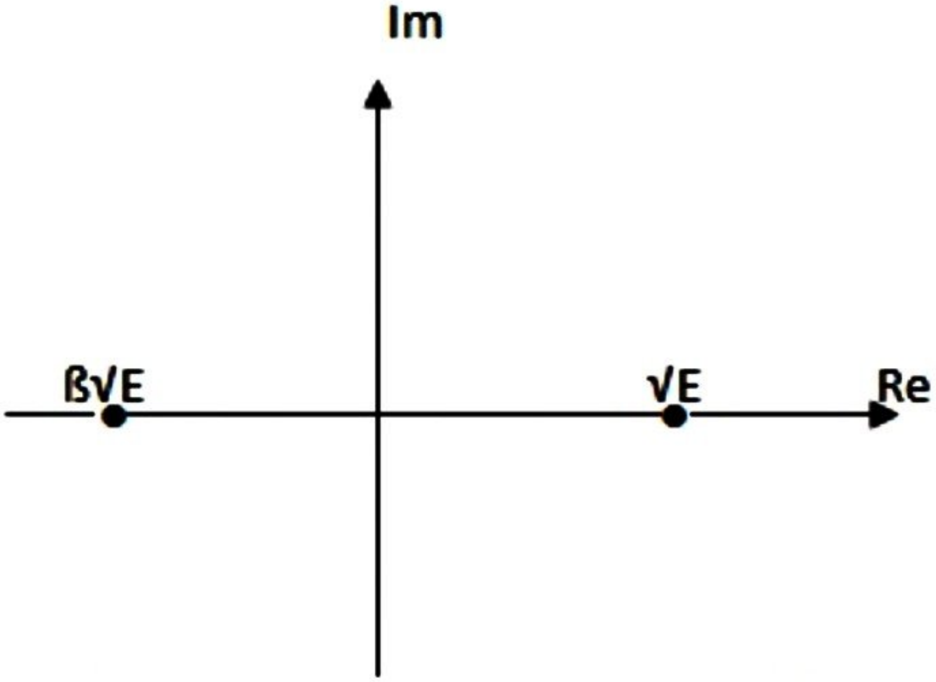
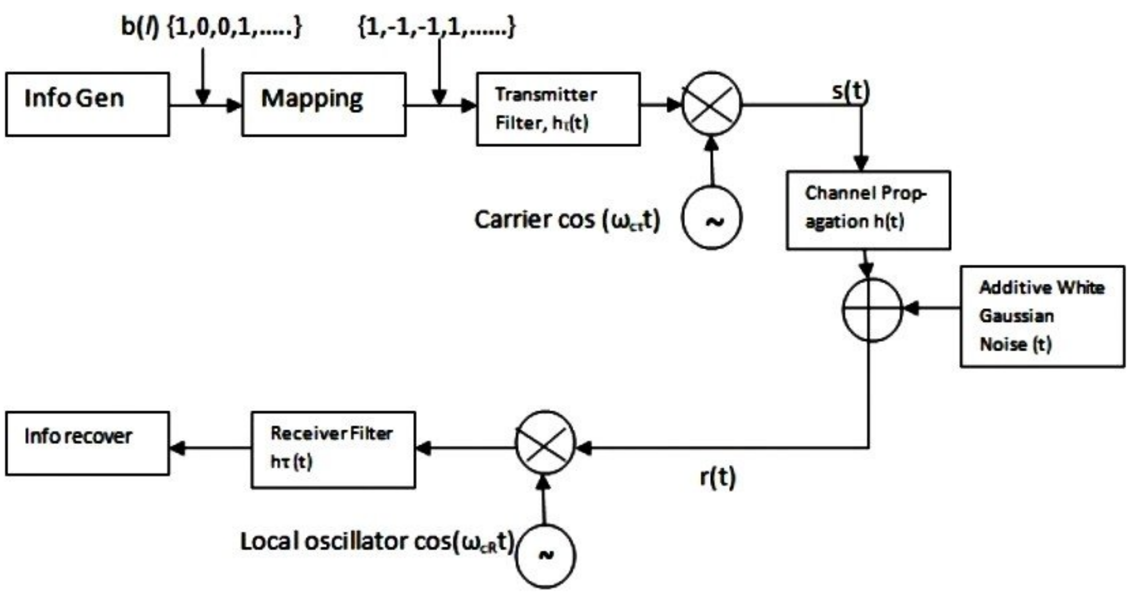


Fig: Constellation Diagram of BPSK



BPSK System Block Diagram

From the Figure, we can describe the BPSK transmission system as 3 parts:

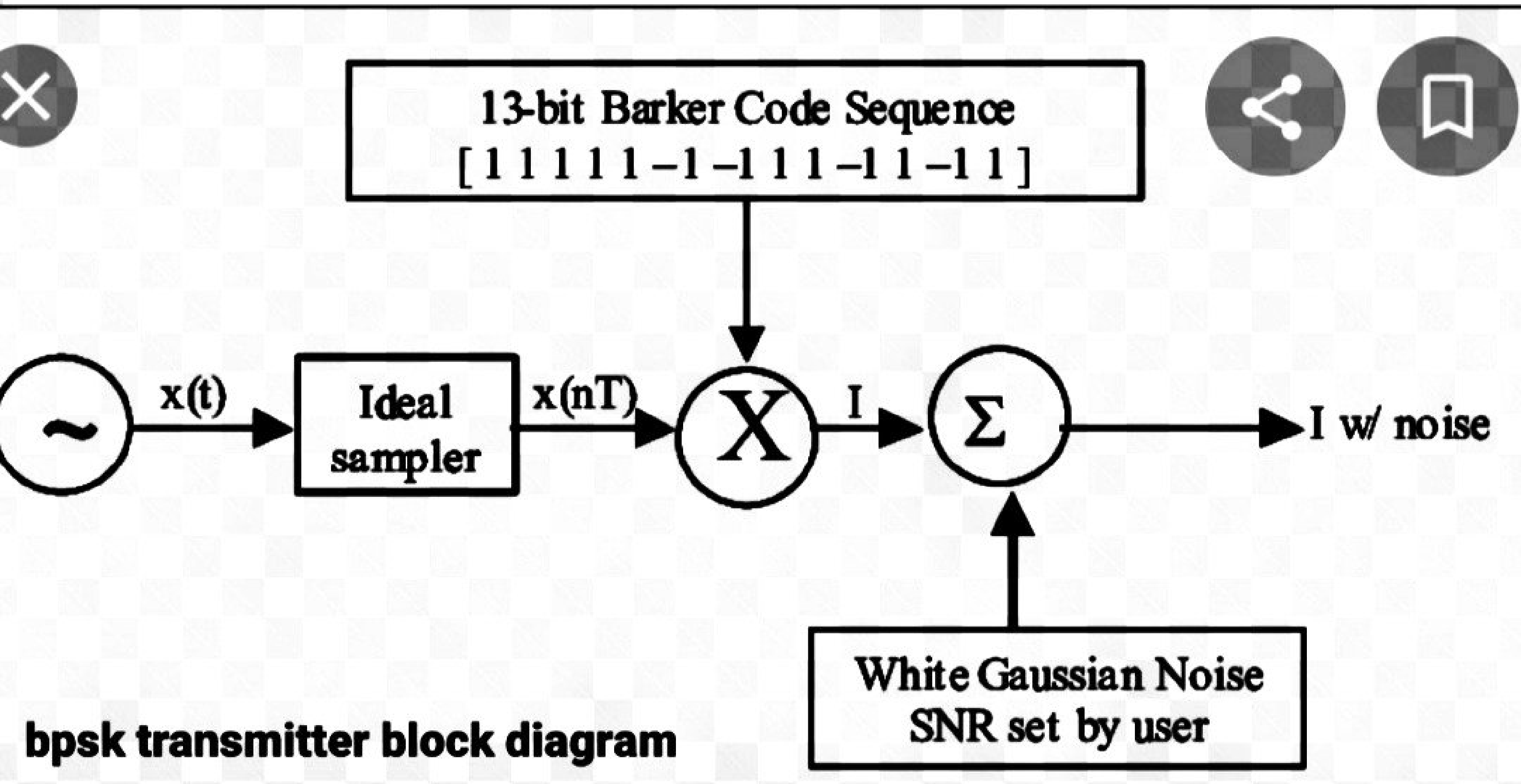
- Transmitter
- Receiver
- AWGN Channel.

Transmitter Part :

Bpsk transmitter

By using the block diagram in Figure above, if we generate, the binary number randomly we will get the information' sequences. Information generated: {b(i)} = 1, 0, 0,.....1.

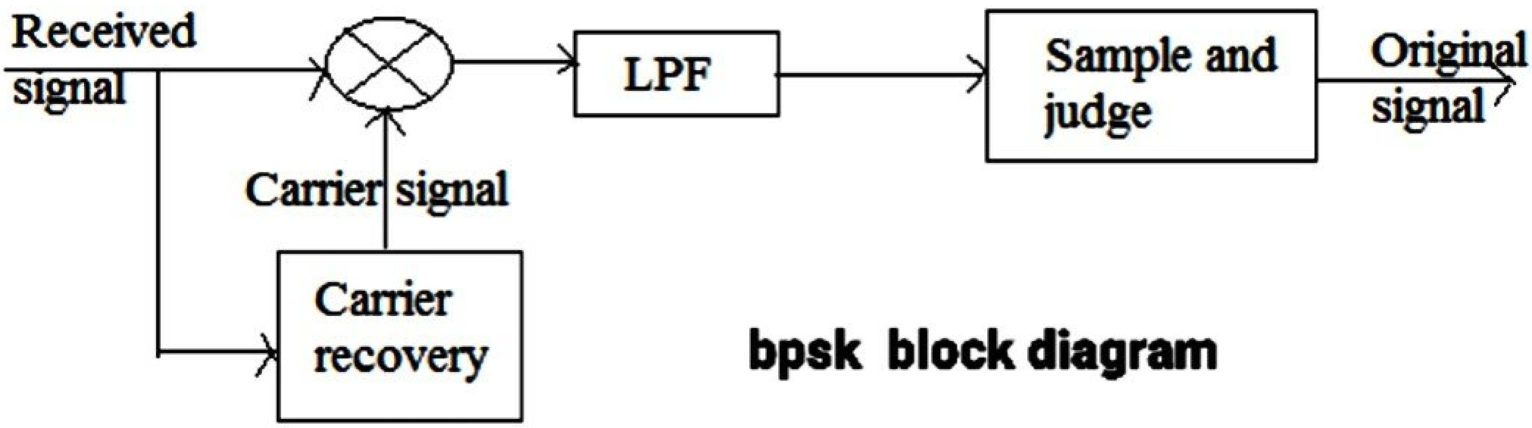
This bit information sequences will modulate the carrier frequency, and the phase of the carrier frequency will shifted as function of binary information.



bpsk transmitter block diagram

The bit information "1" will not shift the carrier phase, and the bit information "0" will shift the carrier phase by 1800 or radians.

BPSK receiver



bpsk block diagram

Receiver Part :

Basically the receiver part as similar with the transmitter part, but the function is contradictive. After demodulate process by using local signal oscillator and filtering by using LPF, the base band signal will be decided.

If the amplitude of the base band signal is < 0 it's decides that a binary "0" was send and if the amplitude of the baseband signal is ≥ 0, the receiver decides that binary "1" was send by the transmitter.

This description is made by assumption that all of the, process of modulator, Band Pass Filter, demodulator and filtering is work perfectly and inter symbol inter fervencies (ISI) is not happen.

qpsk

8.5.1. QUADRIPHASE-SHIFT KEYING (QPSK)

In quadriphase-shift keying (QPSK), the phase of the carrier takes on one of four equally spaced values, such as $\pi/4$, $3\pi/4$, $5\pi/4$ and $7\pi/4$ as shown by

$$S_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right] & 0 \le t \le T \\ 0 & \text{elsewhere} \end{cases} \quad ...(1)$$

where,

i = 1, 2, 3, 4

E - is the transmitted signal energy per symbol

T - is the symbol duration

f_c - carrier frequency equals to $\frac{n_c}{T}$ for some fixed integer n_c.

Each possible value of the phase corresponds to a unique pair of bits called a dibit. For example we may choose the foregoing set of phase values to represent the gray encoded set of dibits 10, 00, 01 and 11.

Using trigonometric identify, we may rewrite equation (1) in the equivalent form of

$$S_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[(2i-1)\frac{\pi}{4}\right] \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left[(2i-1)\frac{\pi}{4}\right] \sin(2\pi f_c t), & \text{for } 0 \le t \le T, \\ 0, & \text{elsewhere} \end{cases} \quad ...(2)$$

1. There are only two orthonormal basis functions, $\phi_1(t)$ and $\phi_2(t)$. The appropriate forms for $\phi_1(t)$ and $\phi_2(t)$ are

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \qquad 0 \le t \le T$$

and

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t) \qquad 0 \le t \le T$$

2. There are four message points and the associated signal vectors are defined by

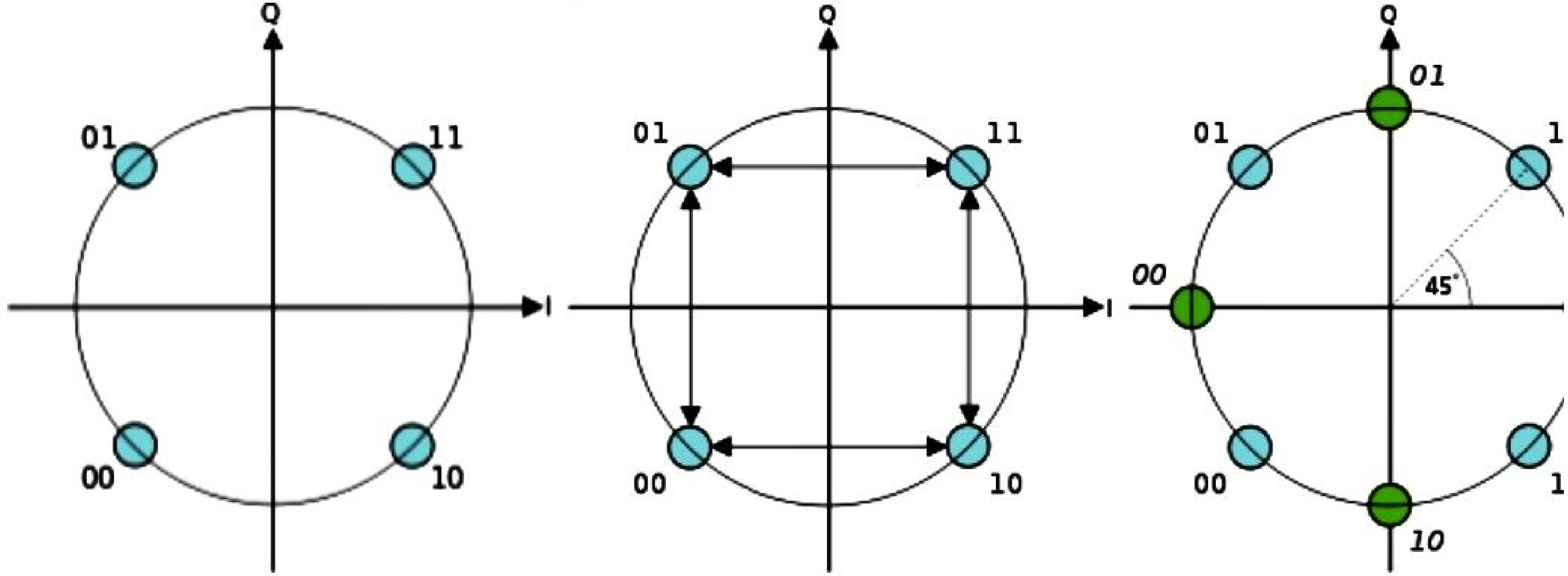
$$S_i = \begin{bmatrix} \sqrt{E} \cos\left[(2i-1)\frac{\pi}{4}\right] \\ -\sqrt{E} \sin\left[(2i-1)\frac{\pi}{4}\right] \end{bmatrix} \qquad i = 1, 2, 3, 4$$

S_{i1} and S_{i2} are called elements of the signal vectors and their values are shown in below table. The first two columns of this table give the associated dibits and phase of the QPSK signal.

Signal space characterization of QPSK

Input dibit	Phase of QPSK signal	Coordinates of message points	
		S_{i1}	S_{i2}
10	$\frac{\pi}{4}$	$+\sqrt{\frac{E}{2}}$	$-\sqrt{\frac{E}{2}}$
00	$\frac{3\pi}{4}$	$-\sqrt{\frac{E}{2}}$	$-\sqrt{\frac{E}{2}}$
01	$\frac{5\pi}{4}$	$-\sqrt{\frac{E}{2}}$	$+\sqrt{\frac{E}{2}}$
11	$\frac{7\pi}{4}$	$+\sqrt{\frac{E}{2}}$	$+\sqrt{\frac{E}{2}}$

qpsk signal constellation



QPSK **uses four points on the constellation diagram, equispaced around a circle**. With four phases, QPSK can encode two bits per symbol, shown in the diagram with Gray coding to minimize the bit error rate (BER) – sometimes misperceived as twice the BER of BPSK.

8.5.3. GENERATION qpsk transmitter

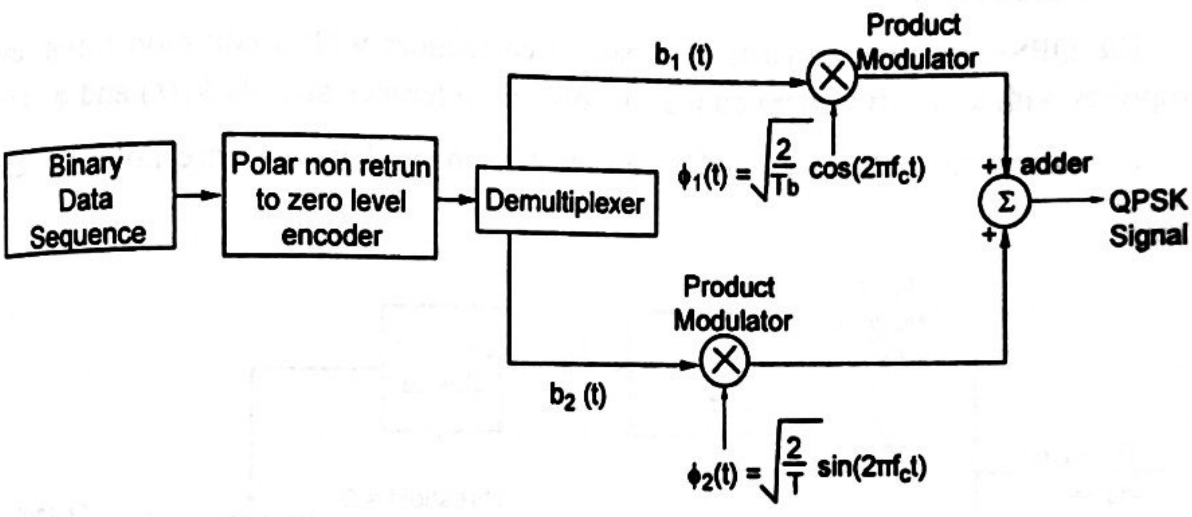


Fig. 8.13. QPSK transmitter

NRZ Encoder

The input binary sequence is first transformed into polar form by a NRZ encoder. Thus symbols 1 and 0 are represented by $+\sqrt{E_b}$ and $-\sqrt{E_b}$.

Demultiplexer

The NRZ encoder output is given to demultiplexer to divide the binary wave into two separate binary waves consisting of the odd and even numbered input bits. These two binary waves are denoted by $b_1(t)$ and $b_2(t)$.

The amplitudes of $b_1(t)$ and $b_2(t)$ equal S_{i1} and S_{i2}, depending on the particular dibit that is being transmitted. The two binary waves $b_1(t)$ and $b_2(t)$ are used to modulate a pair of quadrature carriers or orthonormal basis functions $\phi_1(t)$

equal to $\sqrt{\frac{2}{T}}\cos(2\pi f_c t)$ and $\phi_2(t)$ equal to $\sqrt{\frac{2}{T}}\sin(2\pi f_c t)$. The two binary waves are added to produce the desired QPSK wave.

Symbol Duration

For a QPSK symbol duration is twice as long as the bit duration T_b of the input binary wave (i.e) for a given rate $\frac{1}{T_b}$, a QPSK wave requires half the transmission bandwidth of the corresponding binary PSK wave.

8.5.4. DETECTION

Qpsk receiver

The QPSK receiver consists of a pair of correlators with a common input and supplies with a locally generated pair of coherent reference signals $\phi_1(t)$ and $\phi_2(t)$.

The correlator outputs x_1 and x_2 are each compared with a threshold of zero volts.

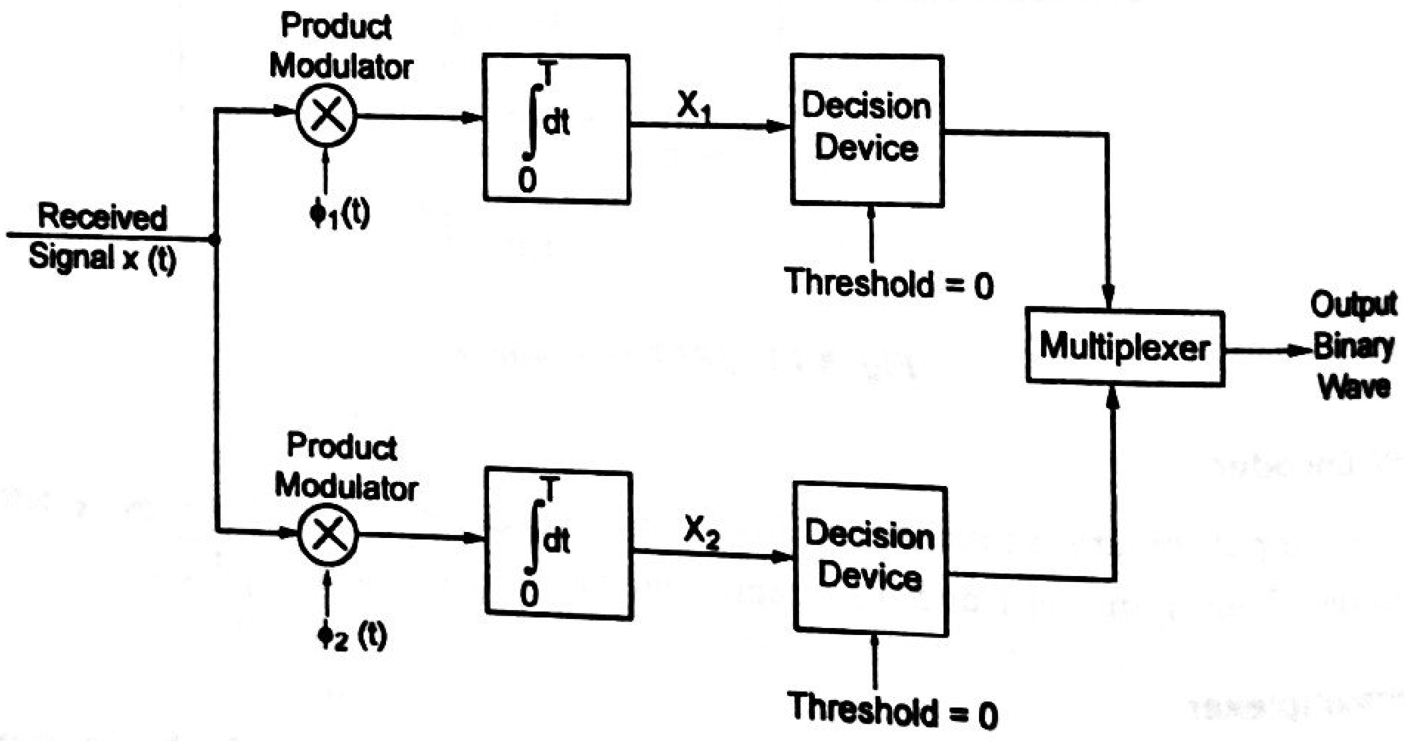


Fig. 8.14. QPSK Receiver

1. If $x_1 > 0$, a decision is made in favor of symbol 1 for the upper or in-phase channel output, but if $x_1 < 0$ a decision is made in favor of symbol 0.
2. If $x_2 > 0$, a decision is made in favor of symbol 1 for the lower or quadrature channel output, but if $x_2 < 0$, a decision is made in favor of symbol 0.

Multiplexer

The two binary sequence x_1 and x_2 are combined in a multiplexer to reproduce the original binary sequence at the transmitter input with the minimum probability of symbol error.

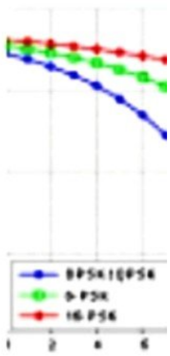
BER VS SNR

What is meant by SNR?

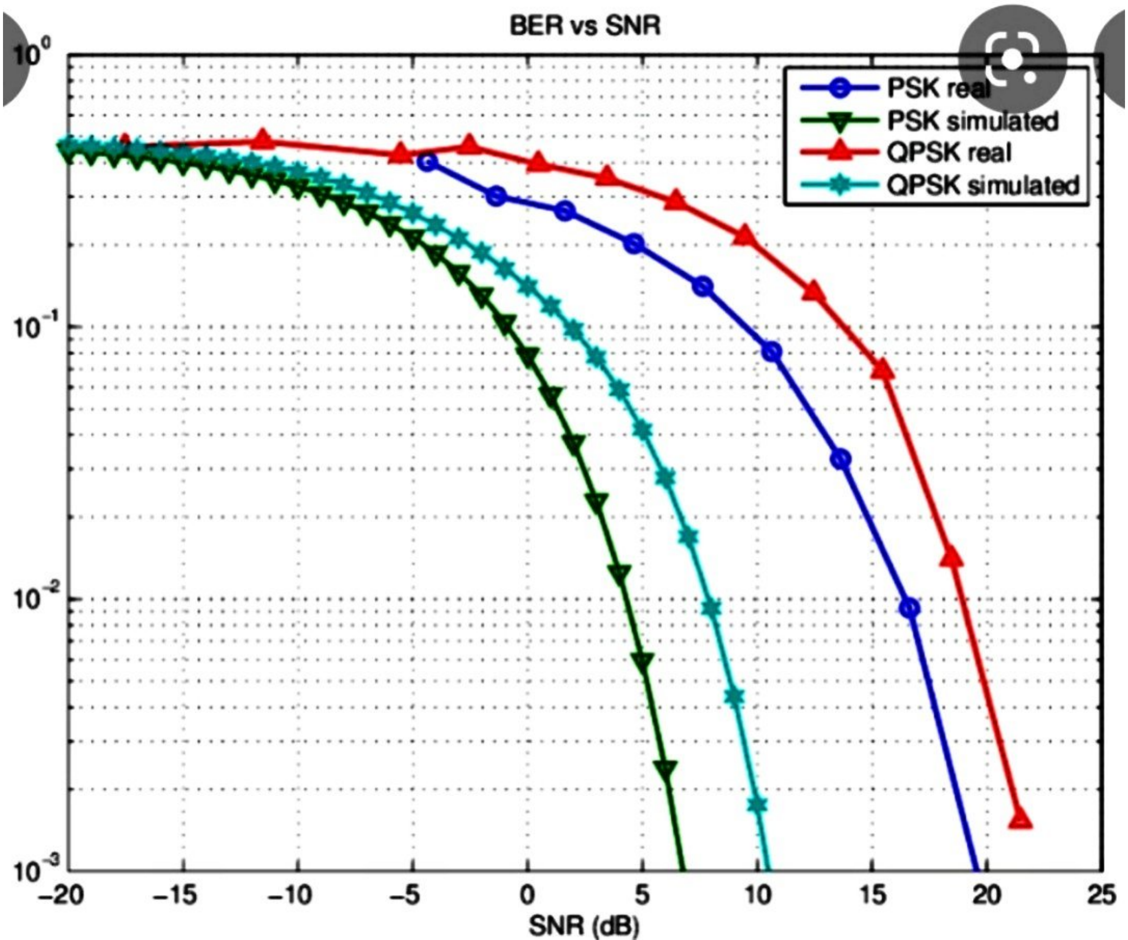
[redacted]atio? In analog and digital communications, a signal-to-noise ratio, often written S/N or SNR, is **a measure of the strength of the desired signal relative to background noise (undesired signal)**.

What is BER plot?

[redacted]e usually plot the BER curves **to describe the performance of a digital communication system**. In optical communication, BER(dB) vs. Received Power(dBm) is usually used; while in wireless communication, BER(dB) vs. SNR(dB) is used.



S/N or SNR, is **a measure of the strength of the desired signal relative to background noise (undesired signal)**.



8.9. QUADRATURE AMPLITUDE MODULATION (QAM)

Quadrature Amplitude Modulation or QAM is a form of modulation which is widely used for modulating data signals onto a carrier used for radio communications. It is widely used because it offers advantages over other forms of data modulation such as PSK, although many forms of data modulation operate along each other.

Quadrature Amplitude Modulation, QAM is a signal in which two carriers shifted in phase by 90 degrees are modulated and the resultant output consists of both amplitude and phase variations. In view of the fact that both amplitude and phase variations are present it may also be considered as a mixture of amplitude and phase modulation.

i.e $\boxed{\text{QAM} = \text{ASK} + \text{PSK}}$

A motivation for the use of quadrature amplitude modulation comes from the fact that a straight amplitude modulated signal, i.e. double sideband even with a suppressed carrier occupies twice the bandwidth of the modulating signal. This is very wasteful of the available frequency spectrum. QAM restores the balance by placing two independent double sideband suppressed carrier signals in the same spectrum as one ordinary double sideband suppressed carrier signal.

quadrature ,constellation

Case 1: M = 16

The signal constellation for QAM consists of a square lattice of message points. Below Figure shows signal constellation for M = 16.

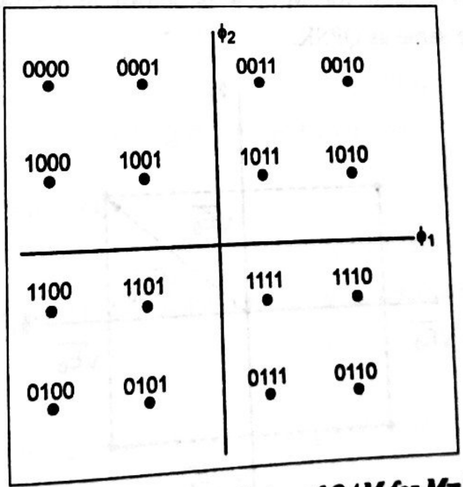


Fig. 8.20. Signal constellation of QAM for M=16

For M = 16, the corresponding signal constellations for the in-phase and quadrature components of the amplitude phase modulated wave are shown below.

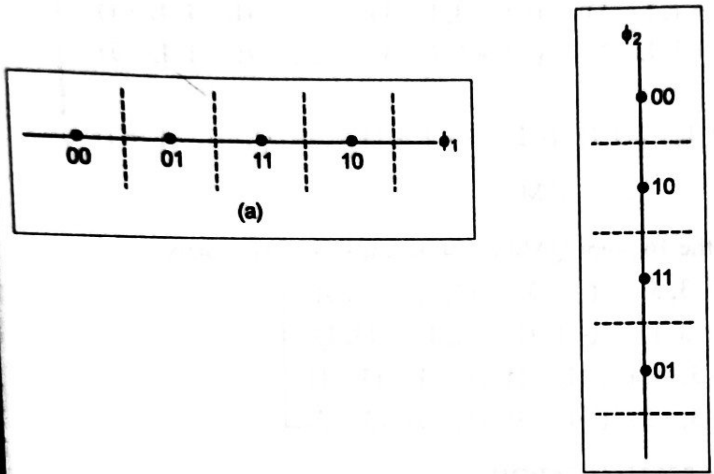
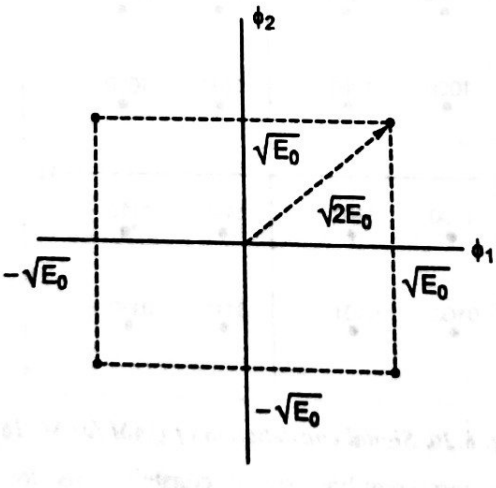


Fig. 8.21. Decomposition of signal constellation of QAM (for M=16) into two signal-space diagrams for (a) in-phase comment $\phi_1(t)$ and (b) quadrature comment $\phi_2(t)$

Case 2: M = 4

The signal constellation for M = 4 is shown in below Figure, which is recognized to be the same as QPSK.

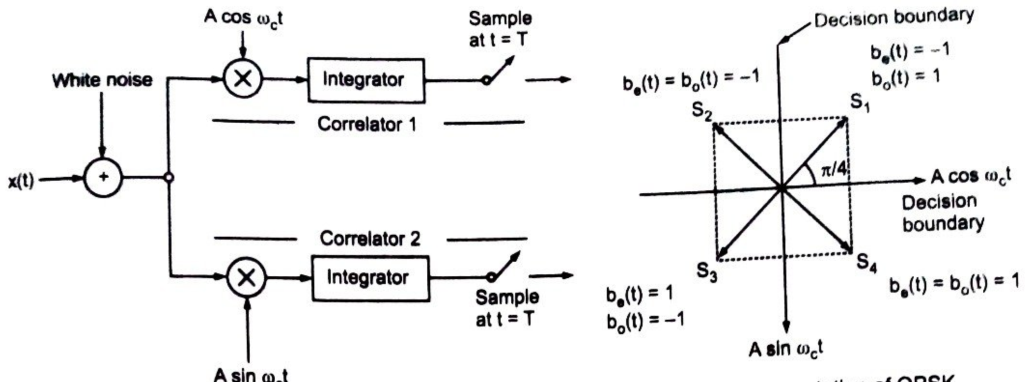


7.19 PROBABILITY OF ERROR OF QPSK SYSTEM

In order to understand the error probability, let us reproduce the QPSK receiver and the phaso diagram of QPSK, as shown in figure 7.32(a) and (b) respectively. From figure 7.32(a). it is obviou that two correlators are required and the locally generated reference waveforms are A cos $\omega_c t$ an sin $\omega_c t$. The received signal plus noise is passed through these correlators to generate the eve and odd bit sequences $b_e(t)$ and $b_o(t)$ as explained earlier. These bit sequences are then adde together to obtain the required message signal as shown in figure 7.32(b).

Let us observe figure 7.32(b) carefully. The reference waveform of correlator 1. i.e.. A cos ω is at an angle $\phi = 45°$ to the axis of orientation of all the four possible signals. Therefore. the axi A cos $\omega_c t$ can be treated as the decision boundary. Note that this decision boundary is at ϕ = 45 therefore. any correlator of the two present can make a mistake if a phase shift of 45° or π radians occurs in the corresponding carrier. Therefore. the probability that correlator 1 o correlator 2 make an error is given by.

$$P_1'(e) = P_2'(e) = \frac{1}{2}\,\text{erfc}\sqrt{\frac{E}{N_0}\cos^2\phi} \quad \ldots (7.120$$



(a) A correlation receiver for QPSK (b) Signal space representation of QPSK

Fig. 7.32.

Substituting $\phi = 45°$ in equation (7.120), we obtain

$$P_1'(e) = P_2'(e) = \frac{1}{2}\,\text{erfc}\sqrt{\frac{E}{N_0}(\cos 45)^2}$$

But, $(\cos 45)^2 = 1/2$

$$\therefore \quad P_1'(e) = P_2'(e) = \frac{1}{2}\,\text{erfc}\sqrt{E/2N_0} \quad \ldots (7.12$$

The probability "P(c)" that the QPSK receiver will correctly identify the transmitted signal equal to the product of the individual probabilities of correct identification of the two correlator

Therefore, Probability of correct reception

$$P(c) = P_1'(c) \times P_2'(c) \quad \ldots (7.12$$

where $P_1'(c)$ = Probability that correlator 1 receives signal correctly.

$P_2'(c)$ = Probability that correlator 2 receives signal correctly.

But $P_1'(c) = 1 - P_1'(e)$ and $P_2'(c) = 1 - P_2'(e)$. Substituting these values into equation (7.1 we get.

$$P(c) = \left[1 - P_1'(e)\right]\times\left[1 - P_2'(e)\right]$$

Therefore, $P(c) = 1 - P'_1(e) - P'_2(e) + P'_1(e)\,P'_2(e)$ ... (7.1

Substitute $P'_1(e) = P'_2(e) = P'(e)$ in equation (7.123), we get

$$P(c) = 1 - 2\,P'(e) + (P'(e))^2 \quad \ldots (7.1$$

Normally. P'(e) is very small. Therefore $P'_2(e)$ is still smaller. Hence, we can neglect the th term on RHS of equation (7.124) to get,

$$P(c) = 1 - 2\,P'(e) \quad \ldots (7.1$$

Hence. the error probability of a QPSK system is given by,

$$P(c) = 1 - P(c) = 1 - [1 - 2\,P'(e)]$$

Therefore. $P(c) = 2\,P'(e)$ ...(7.1

But $P'(e) = P'_1(e) = P'_2(e) = \frac{1}{2}\,\text{erfc}\sqrt{E/2N_0}$ ... From equation (7.126)

Therefore. Error probability of QPSK system = $P(e) = 2\times\frac{1}{2}\,\text{erfc}\sqrt{E/2N_0}$

Therefore. $P(e) = \text{erfc}\sqrt{E/2N_0}$ (7.127

This is the required expression. In equation (7.127) E corresponds to the energy of each

7.15. ERROR PROBABILITY OF BPSK (WITH COHERENT DETECTION)

The various steps to follow in order to obtain the expression for the error probability, are exact same as those followed to obtain error probability of an ASK system.

We know that the BPSK signal is represented as follows:

Binary 1: $x_1(t) = -\sqrt{2P_s}\cos\omega_c t$

Binary 2: $x_2(t) = \sqrt{2P_s}$

Therefore $x_2(t) = -x_1(t)$

We have to use the matched filter for detection of BPSK signal. The expression for erro probability of an optimum filter is given by

$$P_e = \frac{1}{2}\operatorname{erfc}\left[\frac{x_{o1}(T) - x_{o2}(T)}{2\sqrt{2}\,\sigma}\right] \quad \text{... (7.92)}$$

The expression for the signal to noise ratio of a matched filter is given by,

$$\left[\frac{x_{o1}(T) - x_{o2}(T)}{\sigma}\right] = \frac{2}{N_0}\int_{-\infty}^{\infty} |X(f)|^2\, df \quad \text{... (7.93)}$$

Using the Rayleigh's energy theorem, we have

$$\int_{-\infty}^{\infty} |X(f)|^2\, df = \int_{-\infty}^{+\infty} x^2(t)dt = \int_0^T x^2(t)\, dt \quad \text{... (7.94)}$$

The limits of integration of the last term in equation (7.94) are 0 to T because x(t) is presen only over one bit interval T. Substituting equation (7.94) into equation (7.93), we get,

$$\left[\frac{x_{01}(t) - x_{02}(T)}{\sigma}\right]^2_{max} = \frac{2}{N_0}\int_0^T x^2(t)dt \quad \text{...(7.95)}$$

But, $x(t) = x_1(t) - x_2(t)$

and for BPSK, $x_2(t) = -x_1(t)$

Therefore, $x(t) = 2x_1(t) = 2\sqrt{2P_s}\cos\omega_c t$

Substituting this value of x(t) into equation (7.95), we get

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \frac{2}{N_0}\int_0^T 8P_s\cos^2\omega_c t\, dt$$

$$= \frac{16P_s}{N_0}\int_0^T \cos^2\omega_c t\, dt \quad \text{... (7.96)}$$

But $\cos^2\omega_c t = \dfrac{1 + \cos 2\omega_c t}{2}$

Substituting this value of $\cos^2\omega_c t$ into equation (7.96), we obtain

$$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \frac{16P_s}{N_0}\int_0^T \frac{1 + \cos 2\omega_c t}{2}$$

or $$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \frac{8P_s}{N_0}\left[\int_0^T 1\, dt + \int_0^T \cos 2\omega_c t\, dt\right]$$

or $$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \frac{8P_s}{N_0}\left\{[t]_0^T + \frac{1}{2\omega_c}(\sin 2\omega_c t)_0^T\right\}$$

or $$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \frac{8P_s}{N_0}\left\{T + \frac{\sin 2\omega_c T}{2\omega_c}\right\} \quad \text{...(7.97)}$$

The value of second term in the RHS of equation (7.97) is zero,

Therefore, $$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \frac{8P_sT}{N_0} \quad \text{... (7.98)}$$

But $P_s T$ = Energy E.

Therefore, $$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]^2_{max} = \frac{8E}{N_0}$$

Taking the square root of both sides, we have

Therefore, $$\left[\frac{x_{01}(T) - x_{02}(T)}{\sigma}\right]_{max} = \sqrt{\frac{8E}{N_0}} \quad \text{... (7.99)}$$

Substituting this expression into equation (7.99), we get the error probability for BPSK as under:

$$P_e = \frac{1}{2}\operatorname{erfc}\left[\frac{1}{2\sqrt{2}}\cdot\sqrt{\frac{8E}{N_0}}\right]$$

Therefore, $$P_e = \frac{1}{2}\operatorname{erfc}\left[\sqrt{\frac{E}{N_0}}\right] \quad \text{... (7.100)}$$

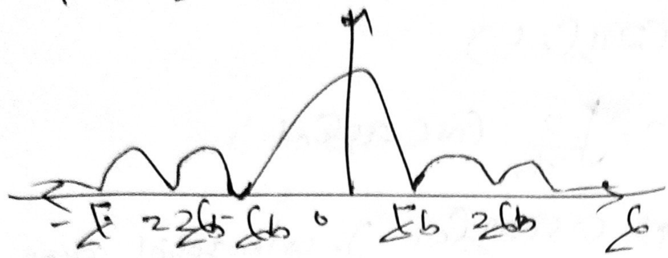
This is the expression for error probability of BPSK with matched filter receiver.

This is the expression for bit error probability P_B.

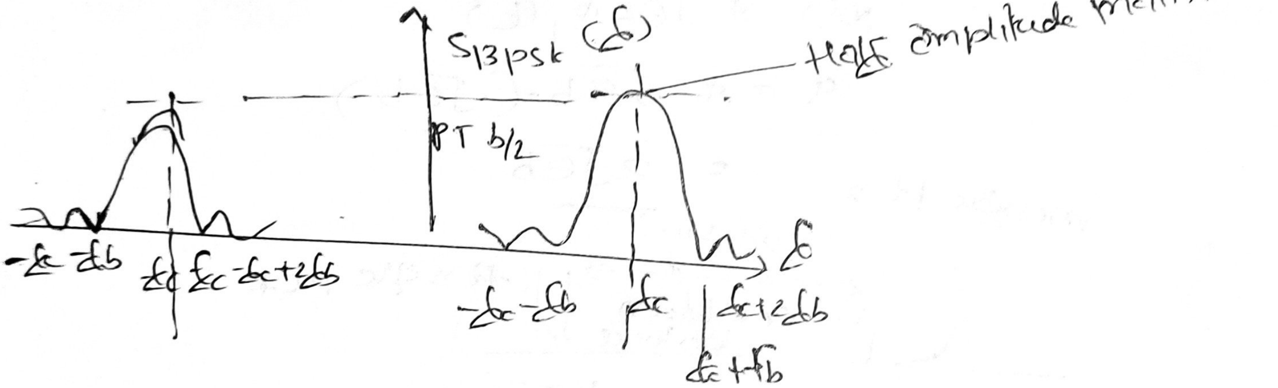
It indicates that the

Spectrum of BPSK signals

Psd of NRZ baseband signal



Psd of BPSK signal



$$X(f) = V_b T_b \frac{\sin(\pi f T_b)}{\pi f T_b}$$

$$\text{Psd } S(f) = \frac{|X(f)|^2}{T_s}$$

$$= \frac{V_b^2 T_b^2}{T_s}\left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

$$S(f) = V_b^2 T_b \left[\frac{\sin(\pi f T_b)}{\pi f T_b}\right]^2$$

$$S_{BPSK}(f) = V_b^2 T_b \left\{ \frac{1}{2}\left[\frac{\sin\pi(f_c - f)T_b}{\pi(f_c - f)T_b}\right]^2 + \frac{1}{2}\left[\frac{\sin\pi(f_c + f)T_b}{\pi(f_c + f)T_b}\right]^2 \right.$$

~~Modulated signal~~

~~$s(t) = \pm \ldots 2 \ldots \cos(2\pi f_c t)$~~