24/6/2016

Module I :-

Introduction to Digital Communication :-

Digital communication deals with the transmission of information in digital form from a source that generates the information to one or more destinations. In the design of digital communication system the information source, communication channel and information sink are specified. The challenge is to design the transmitter and receiver with following guidelines

- Encode / modulate the message signal generated by the source of information, transmit it over channel and produce an estimate of it at the receiver output that satisfies the requirements of the end user.
- Do all this at an affordable cost.

Elements of a digital Communication

The functional blocks of the transmitter are

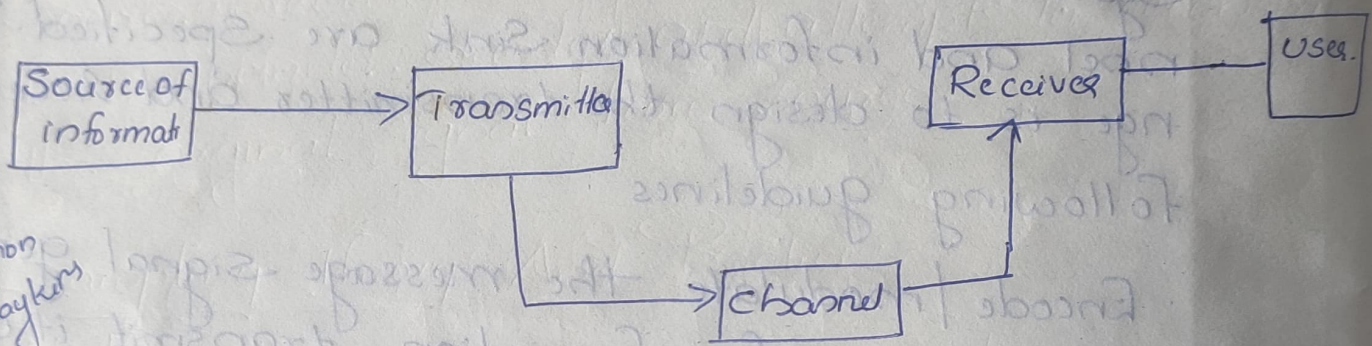
- Source encoder - decoder.
- Channel encoder - decoder.
- modulator - demodulator.

The source encoder removes redundant information from message signal and is responsible for efficent use of channel.

The resulting sequence of symbol is called the Source codeword. The data stream is processed next by channel encoder, which produces a new sequence of symbols called channel codeword. Finally the modulator represent each symbol of codeword as signals.

Block Diagram of Digital Communication System:-

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11. Simon Haykin

Elements of Communication System.

The source output may be either an analog signal, such as audio or video signal or a digital signal. In digital communication system, the message produced by the source are converted into sequence of binary digits. Representation should be with little or no redundancy. The process of efficently converting the output of source into sequence of binary digits is called source encoding.

This Sequence called information Seq
is passed to channel encoder. The purpose
channel encoder is to introduce in controlled
manner, Some redundancy in the binary infor
mation Sequence that can be used at the recei-
ver to overcome the effects of noise and inter
ferance encountered in the transmission of
Signals. Thus the added redundancy Serves to
increase the reliability of received data and
improves the fidelity of received Signals.

The binary Sequence at the output of
channel encoder is passed to digital modulator
which Serves as interferance to communicatio
channel. The primary purpose of digital modulator
is to map the binary information sequence
into Signal waveform.

The communication channel is the physical
medium that is used to Send the Signal from tr-
ansmitter to receiver. Whatever the physical
medium used for transmission of information
the essential feature is that the transmitted
Signal is corrupted in a random manner by
a variety of possible mechanisms Such as
additive thermal noise generated by electronic
devices.

At the receiving end of digital communi-
cation System, the digital demodulator processes
the channel corrupted transmitted waveform
and reduces the waveform to a sequence of

numbers that represent estimates of the transmitted data symbols. This sequence of numbers is passed to the channel decoder, which attempts to reconstruct the orginal information sequence from knowledge of code used by channel encoder and redundancy contained in received data.

As a final step, when analog output is desired source decoder accepts the output sequence from channel decoder and attempts to reconstruct the orginal signal from source.

Sampling Theory And Practice:-

27/6/16

Need:-

There are two types of signals

1. Continous time Signal.
2. Discrete time Signal.

Due to recent advancement in technology, the processing of discrete time signals is more flexible and preferable than continous time signal. This means that in practice, although we have a large number of continous signals we prefer processing of discrete time signals. For this purpose we should be able to convert a continous time signal into discrete time signal. For conversion a fundamental mathematical tool known as sampling theorem is used.

Importance:-

A continous time signal may be completely represented and recovered from the knowledge of samples taken uniformly, with the help of Sampling theorem.

Sampling theorem may be viewed as a bridge between continous and discrete time signals.

It provides a mechanism for representing a continous time signal by a discrete time signal.

Frequency Domain Description of Sampling:-

Consider an arbitary signal $g(t)$ of finite energy for all time t. Suppose we Sample $g(t)$ at a uniform rate once every T_s seconds. Consequently we obtain an infinite sequence of samples spaced T_s seconds apart denoted by $\{g(nT_s)\}$, where 'n' takes possible integer values.

Here T_s is Sampling period and

$f_s = 1/T_s$ is sampling rate.

Let $g_s(t)$ denote the signal obtained by indidually weighting the elements of a periodic Sequence of delta function spaced T_s apart.

$$g_s(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t-nT_s)$$

$g_s(t)$ is called ideal sampled signal.

Taking Fourier transform on both sides

$$G_s(f) = \sum_{n=-\infty}^{\infty} g(nT_s)\, e^{-j2\pi nfT_s}$$

Suppose if the signal is band limited with no higher frequency components higher than W Hz. If the sampling period $T_s = 1/2W$

$$G_s(f) = \sum_{n=-\infty}^{\infty} g(n/2W) \exp\left(-\frac{j\pi nf}{W}\right)$$

$$G_s(f) = f_s G(f) + f_s \sum_{m=-\infty}^{\infty} G(f-mf_s)$$

Suppose

$G(f) = 0 \quad |f| \geq W$

$f_s = 2W$

$$G(f) = \frac{1}{2W} G_s(f) \quad -W < f < W$$

$$\therefore G(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) e^{-j\pi nf/W} \quad -W < f < W$$

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Reconstruction:-

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{-j2\pi ft} \cdot df$$

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{1}{2W} \int_{-W}^{W} e^{-j2\pi f\left(t - \frac{n}{2W}\right)} \cdot df$$

$$= \frac{1}{2W} \int_{-W}^{W} e^{-j2\pi f\left(t - \frac{n}{2W}\right)} \cdot df$$

$$= \frac{\sin\left(2\pi Wt - n\pi\right)}{2\pi Wt - n\pi}$$

$$= \text{sinc}\left(2Wt - n\right).$$

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \text{sinc}\left(2Wt - n\right) \quad -\infty < t < \infty.$$

This is desired reconstruction formula.

Sampling Theorem:-

Sampling theorem states that:

A band limited signal of finite energy, which has no frequency components higher than fm Hz, may be completely recovered from the knowledge of its samples taken at the rate of 2fm samples per second

Proof :-

Consider a continous signal $x(t)$ whose spectrum is band limited to f_m Hz.

$\therefore X(\omega)$ is zero for $|\omega| > \omega_m$.

ie $X(\omega) = 0$ for $|\omega| > \omega_m$

where $\omega_m = 2\pi f_m$

Sampling of $x(t)$ at a rate of f_s Hz may be achieved by multiplying $x(t)$ by an impulse train $\delta_{Ts}(t)$. Impulse consist of unit impulses repeating periodically every T_s seconds where $T_s = 1/f_s$. This multiplication results in sampled signal $g(t)$.

The resulting signal may be written as

$$g(t) = x(t)\,\delta_{Ts}(t) \longrightarrow (1)$$

Trignometric Fourier Series expansion of impulse train $\delta_{Ts}(t)$ is expressed as

$$\delta_{Ts}(t) = \frac{1}{Ts}\left[1 + 2\cos\omega_s t + 2\cos 2\omega_s t + \ldots\right] \longrightarrow (2)$$

$$\omega_s = 2\pi f_s = \frac{2\pi}{Ts}$$

Putting (2) in (1)

$$g(t) = \frac{1}{Ts}\left[x(t) + 2x(t)\cos\omega_s t + 2x(t)\cos 2\omega_s t + \ldots\right] \longrightarrow (3).$$

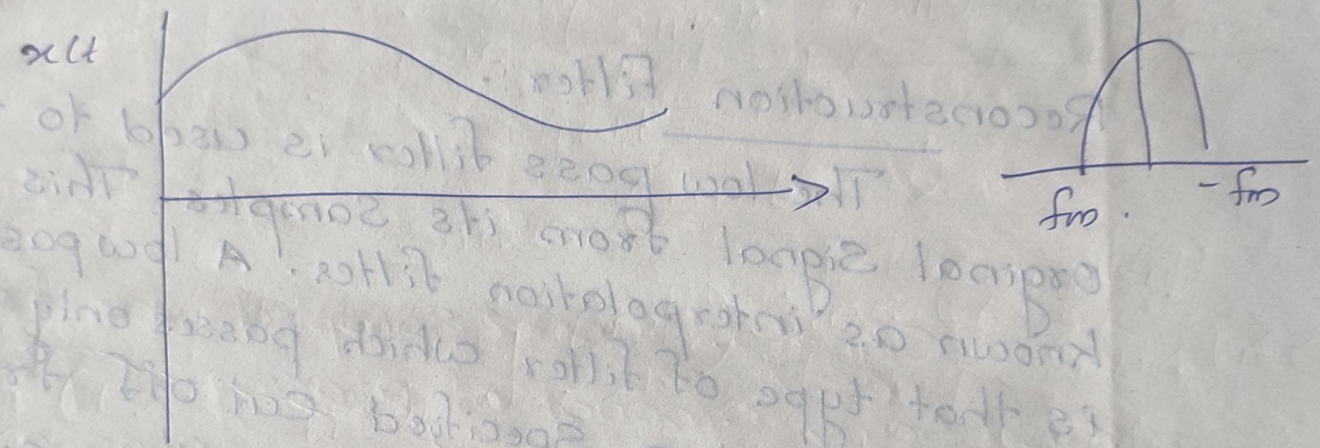
Fourier transform of $g(t)$ is $G(\omega)$

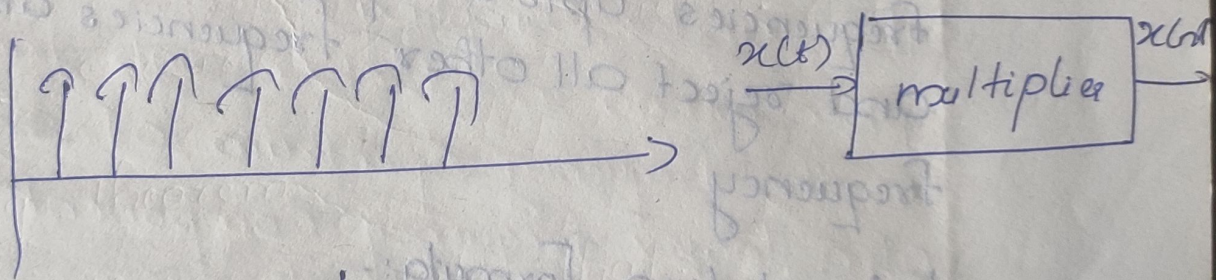
$$G(\omega) = \frac{1}{Ts}\left[X(\omega) + X(\omega-\omega_s) + X(\omega+\omega_s) + X(\omega-2\omega_s) + \ldots\right]$$

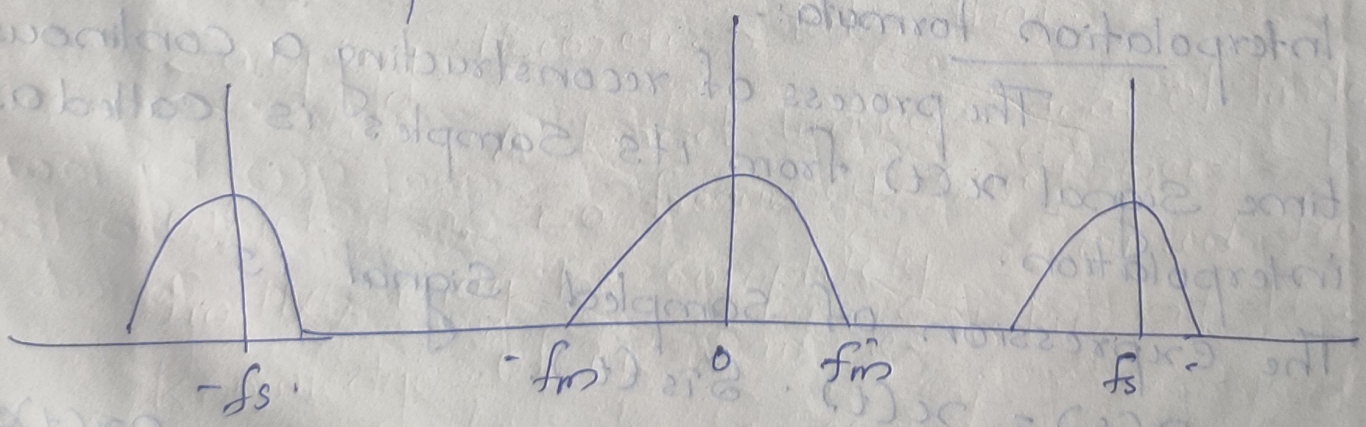
$$G(\omega) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$$

To reconstruct $x(t)$ from $g(t)$, we must be able to recover $X(\omega)$ from $G(\omega)$. This is possible if there is no overlap between successive cycles of $G(\omega)$.

$$f_s > 2f_m$$







Nyquist Rate And Nyquist Interval:-

When sampling rate becomes exactly equal to 2fm samples per second, then it is called Nyquist rate. Nyquist rate is also called minimum sampling rate.

Maximum sampling interval is called Nyquist interval.

$$f_s = 2f_m$$

Reconstruction Filter:-

The low pass filter is used to recover orginal signal from its samples. This is also known as interpolation filter. A low pass filter is that type of filter which passes only low frequencies upto a specified cut off frequency and reject all other frequencies above cut off frequency.

Interpolation Formula:-

The process of reconstructing a continous time signal $x(t)$ from its samples is called as interpolation.

The expression of sampled signal is

$$g(t) = x(t) \cdot \delta_{Ts}(t).$$

$$g(t) = \frac{1}{T_s}\left[x(t) + 2x(t)\cos\omega_s t + 2x(t)\cos \omega_s t + \ldots\ldots\right].$$

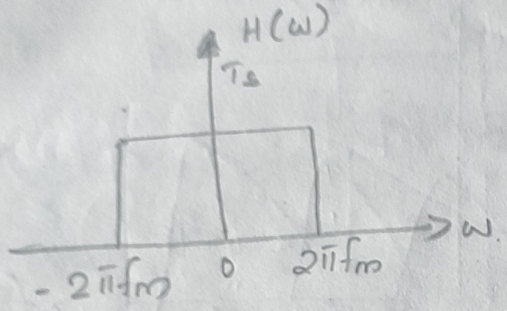
To recover the sampled signal is passed throug ideal low pass filter.

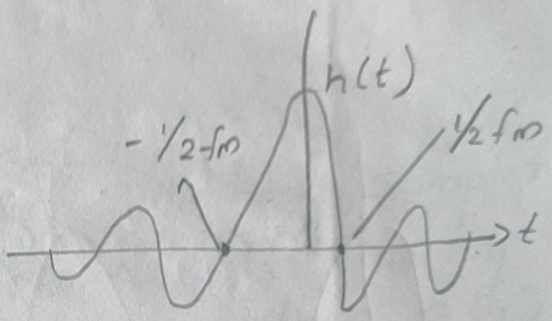
$$H(\omega) = T_s \, rect\left(\frac{\omega}{4\pi f_m}\right)$$

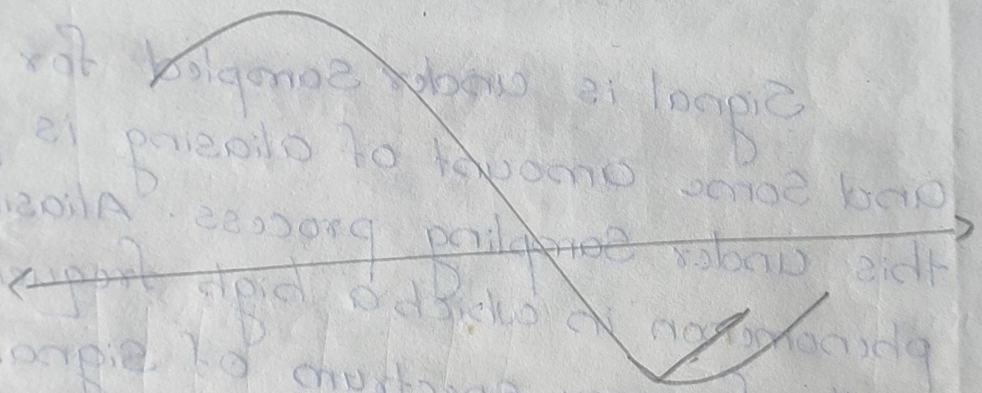
$$h(t) = F^{-1}[H(\omega)] = F^{-1}\left[T_s \, rect\left(\frac{\omega}{4\pi f_m}\right)\right]$$

$$h(t) = 2 f_m T_s \, sinc(2\pi f_m t)$$

$$T_s = \frac{1}{2 f_m}$$







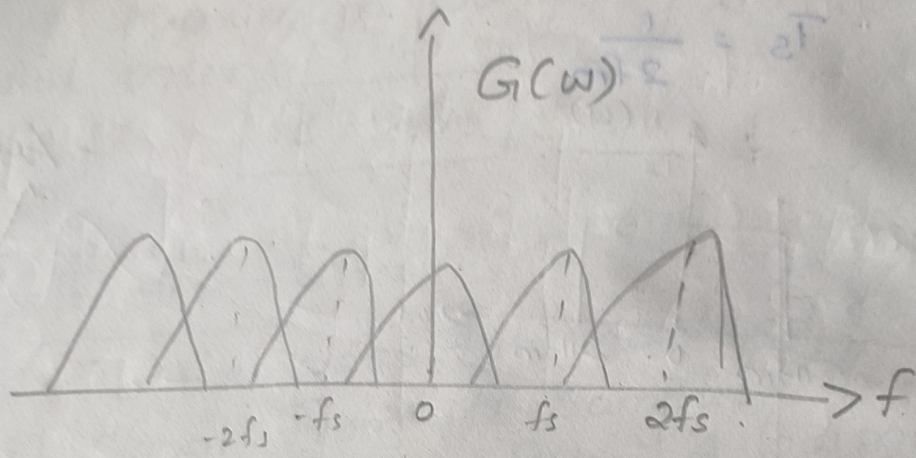
$$x(t) = \sum_k x(kT_s)\, h(t - kT_s)$$

$$x(t) = \sum_k x(kT_s)\, sinc(2\pi f_m - k\pi)$$

is known as interpolation formula.

29/6/16 · Practical Sampling And Aliasing:-

When a continous time band limited signal is sampled at a rate lower than Nyquist rate $f_s < 2f_m$, then the successive cycles of the Spectrum $G(\omega)$ of sampled signal $g(t)$ overlap with each other.



Signal is under sampled for $f_s < 2f_m$ and some amount of aliasing is produced in this under sampling process. Aliasing is the phenomenon in which a high frequency component in the frequency spectrum of signal takes identity of a lower frequency component in the spectrum of sampled signal.

It is not possible to recover orginal signal $x(t)$ from sampled signal $g(t)$ by low pass filtering since spectral components in the overlap region adds and signal is distorted.

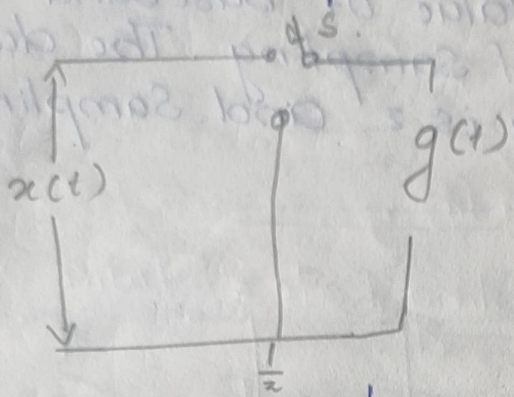
To avoid aliasing
Pre alias filter must be used to limit the band frequencies to f_m

$f_s > 2f_m$

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Ideal Sampling:-

In this sampling, the sampling function is a train of impulses.



This circuit is known as switching sampler. Circuit consist of a switch. Now if closing time 't' of switch approach zero, then g(t) will contain only instantaneous value of input signal x(t)

We know that train of impulses may be represented as

$$\delta_{T_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$g(t) = x(t) \cdot \delta_{T_s}(t).$$

$$= x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

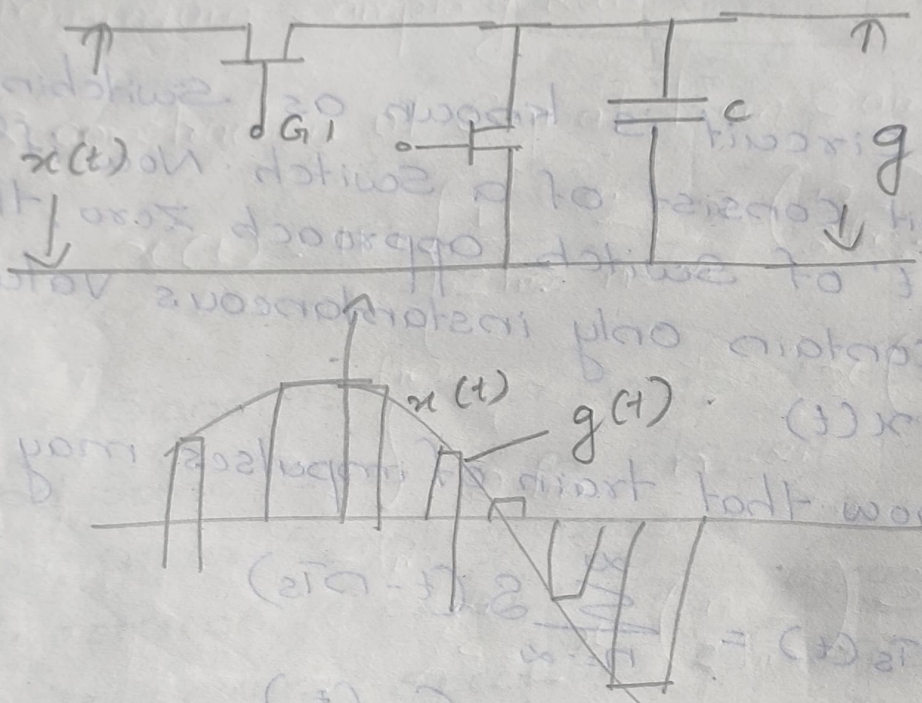
$$= \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \delta(t - nT_s)$$

Fourier Transform

$$G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

Flat Top Sampling :-

In flat top sampling, the top of Samples remains the same and is equal to instantaneous value of baseband Signal $x(t)$ at the start of sampling. The duration or width of Sample is τ and Sampling rate $f_s = 1/T_s$.



Starting edge of pulse represent the point where base band Signal is sampled and width is determined by function $h(t)$

$$g(t) = s(t) \circledast h(t).$$

$$g(t) = \int_{-\infty}^{\infty} s(\tau)\, h(t-\tau)\, d\tau.$$

$$g(t) = \int_{-\infty}^{\infty} \sum x(nT_s)\, \delta(\tau - nT_s)\, h(t-\tau)\, d\tau$$

$$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \int_{-\infty}^{\infty} \delta(z - nT_s) h(t-z)\, dz.$$

$$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s) h(t - nT_s).$$

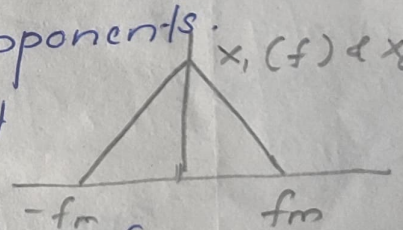
$$G(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) H(f).$$

Sampling Theorem for Band Pass Signals:

For bandpass Signals there is a different criteria to Sample the signal. Sampling theorem for bandpass signal is

The bandpass Signal whose maximum bandwidth is $2f_m$ can be completely represented into and recovered from Samples if it is sampled at a rate of twice the bandwidth.

The bandpass Signal is represented as interms of inphase and quadrature components.



Let $x_i(t)$ = Inphase component of $x(t)$.

$x_q(t)$ = Quadrature Component of $x(t)$.

$$x(t) = x_i(t)\cos(2\pi f_c t) - x_q(t)\sin(2\pi f_c t)$$

Inphase and quadrature components are obtained by multiplying $x(t)$ by $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ and then supressing the sum frequencises by lowpass filters

Reconstruction formula is

$$x(t) = \sum_{n=-\infty}^{\infty} x\left(\frac{n}{4f_m}\right) \text{Sinc}\left(2f_m t - \frac{n}{2}\right) \cos\left[2\pi f_c \left(t - \frac{n}{4f_m}\right)\right]$$

$$x\left(\frac{n}{4f_m}\right) = x(nT_s)$$

$$T_s = 1/4f_m.$$

Minimum Sampling Rate = Twice of Bandwidth

Reconstruction of Signal From Samples:-

$$x(t) = \text{IFT}\left\{\frac{1}{2f_m} \sum_{n=-\infty}^{\infty} x\left(\frac{n}{2f_m}\right) e^{-j\pi f/f_m}\right\}$$

$$= \int_{-f_m}^{f_m} \frac{1}{2f_m} \sum_{n=-\infty}^{\infty} x\left(\frac{n}{2f_m}\right) e^{-j\pi f/f_m} e^{j2\pi ft}\, df.$$

$$= \sum_{n=-\infty}^{\infty} x\left(\frac{n}{2f_m}\right) \frac{\sin\pi(2f_m t - n\pi)}{\pi(2f_m t - n\pi)}$$

$$x(t) = \sum_{n=-\infty}^{\infty} x\left(\frac{n}{2f_m}\right) \text{Sinc}(2f_m t - n)$$

Spacing between Successive Samples is $\frac{1}{2f_m}$ s.

$f_s = 2f_m$ Samples / second.

Waveform Coding :-

Need:- Waveform coding technique is used to convert analog PAM signal into digital signal. This digital signal will be in the form of zeros and ones.

Concept of Quantisation:-

To convert an analog signal to digital signal following steps are required:

- Get the samples of signal according to sampling theorem.
- Mark time instants t_0, t_1, t_2
- At each time instants, the magnitude of the signal is measured and thus samples of signals are taken.
- Now signal is a discrete function and the magnitudes are continous function ie signal is continous.

To convert the magnitude, we use quantisation

The total amplitude range which the signal may occupy is divided into number of standard levels. Amplitude of $x(t)$ lie in the range of $(-m_p, m_p)$ which is partioned into L levels, each of magnitude

$$\Delta v = \frac{2m_p}{L}$$

Now each Sample is approximated or rounded off to nearest quantised level.

Pulse Code Modulation: [PCM]

PCM is a digital pulse modulation technique

Elements of PCM:

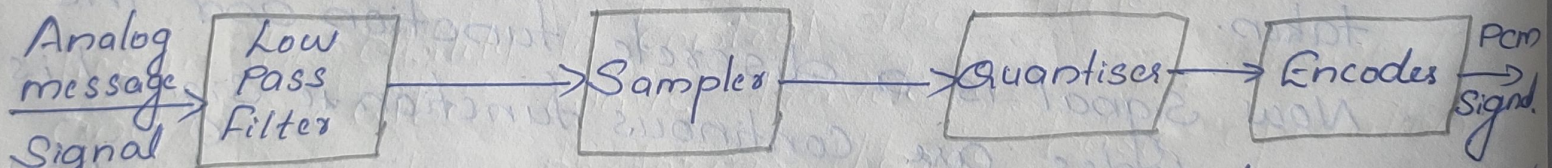
It consist of three main parts:

- Transmitter
- Transmission path.
- Receiver

Transmitter:-

Essential operation of transmitter are

- Sampling
- Quantising
- Encoding.



Specifically transmitter consist of two components

Pulse Amplitude modulator

Analog to digital Converter.

Sampling in Transmitter:-

The incoming Signal is sampled with train of rectangular pulses short enough to closely approximate the instantaneous sampling process. A low pass aliasing filter is used at the front end of

PAM to exclude frequencies greater than w bef Sampling. The application of sampling permits the reduction of continously varying message signal to a limited number of discrete values per second.

2/7/16 Quantisation in Transmitter :-

PAM representation of message signal is then quantised in the ADC, thereby providing new representation of signal that is discrete both in time domain and frequency domain. Quantisation process may follow a uniform procedure. Assuming memoryless quantisation, the use of non uniform quantiser is equivalent to passing a message signal through compressor and then apply to a uniform quantiser. A particular form of compression law called μ-law is used

$$|v| = \frac{\ln[1+\mu|m|]}{\ln[1+\mu]}$$

m and v are i/p and o/p voltages.
$\mu \rightarrow$ +ve constant.

$$\frac{d|m|}{d|v|} = \frac{\ln(1+\mu)}{\mu}(1+\mu|m|)$$

Another Compression law A law

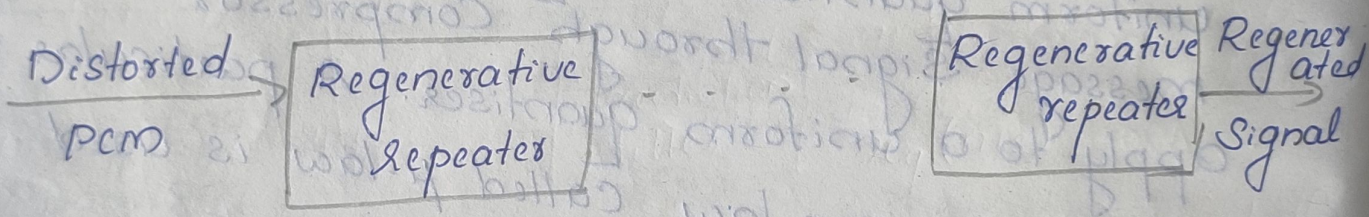
$$|v| = \begin{cases} \frac{A|m|}{1+\ln A} & 0 \le |m| \le \frac{1}{A} \\ \frac{1+\ln\left(\frac{|m|}{|m|}\right)}{1+\ln A} & \frac{1}{A} \le |m| \le 1 \end{cases}$$

To restore the signal samples we use a device complementary to compressor ie expander. The cascade combination of compressor and expander is compander.

Encoding:-

Through sampling and quantisation, the specification of analog signals becomes discrete levels. We require encoding process to translate the discrete set of samples to more appropriate form of signal.

Transmission Path:-



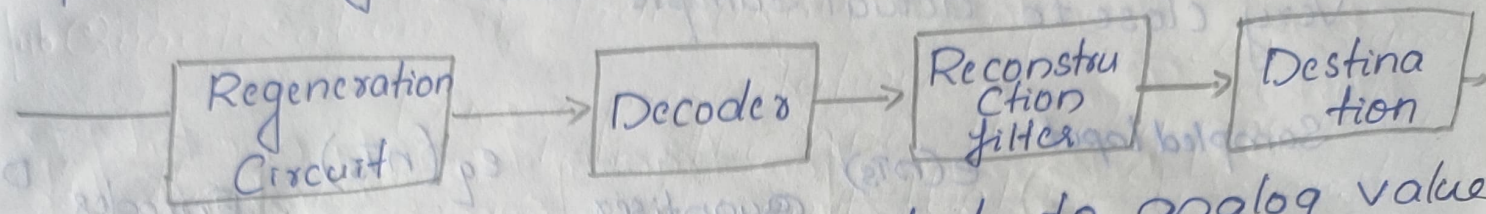
The most important feature of PCM system lies in the ability to control the effects of distortion and noise when wave travels through channel. The regenerative performs three basic operation

- Equaliation
- Timing
- Decision.

Hence each repeater actually reproduces clean noise free PCM signal from distorted signal. This improves performance of PCM in presence of noise.

04/7/16 PCM Receiver:-

The regenerator at start reshape the pulse and remove the noise. The signal is converted into parallel digital words for each sample.



Digital word is converted to analog value with help of sample and hold circuit.

Advantages:-

- high noise immunity.
- can store PCM signal.
- use various coding techniques.

Disadvantages:-

PCM is complex.
Requires large band width.

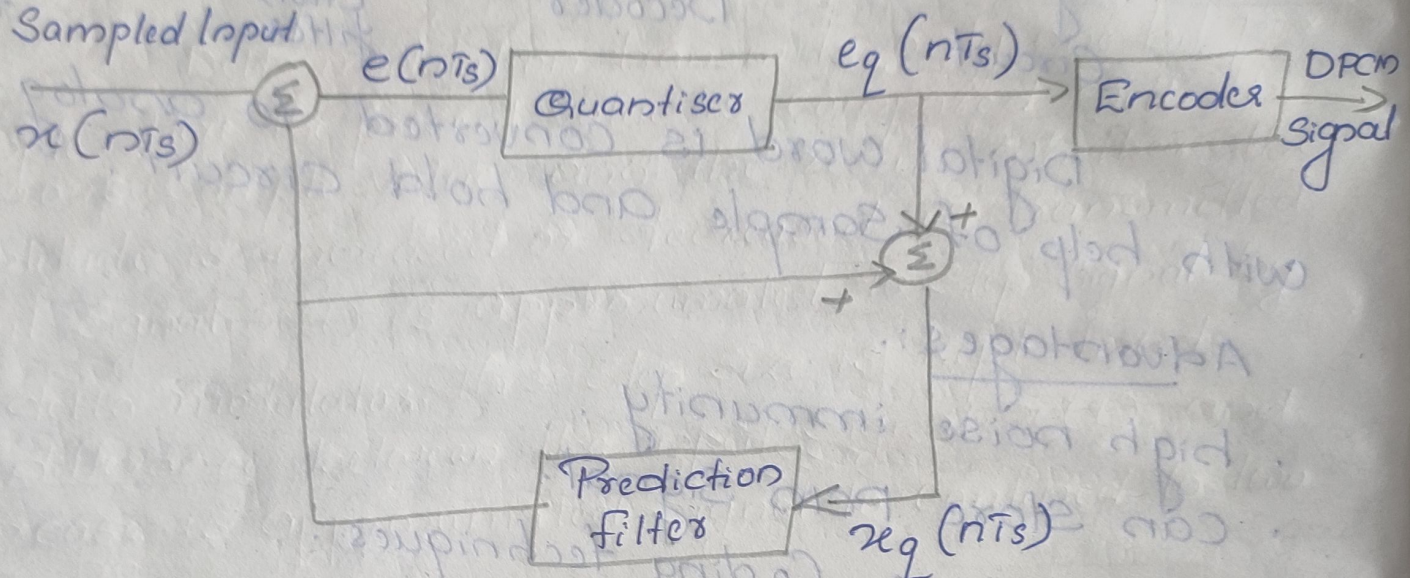
Differential Pulse Code Modulation:- [DPCM]

05/7/16 Reason to use:-

Samples of signals are highly correlated to each other. Signal does not change fast. This means that its value from present sample to next sample does not differ by large amount. When these signals are sampled and encoded there will be Redundant information.

Working Principle :-

DPCM works on the principle of prediction. The value of present sample is predicted from past samples. The prediction may not be exact but it is very close to actual sample value.



The sampled signal is denoted by $x(nT_s)$ and the predicted signal by $\hat{x}(nT_s)$. The comparator finds out the difference between the actual sample value $x(nT_s)$ and predicted sample value $\hat{x}(nT_s)$. This is known as prediction error.

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

Thus, error is difference between unquantised sample $x(nT_s)$ and prediction of it $\hat{x}(nT_s)$. This signal is called $x_q(nT_s)$.

Number of bits are reduced.

Quantizer output can be written as

$$e_q(n\bar{T}s) = e(n\bar{T}s) + q(n\bar{T}s)$$

Here, $q(n\bar{T}s)$ is quantisation error.

$$x_q(n\bar{T}s) = \hat{x}(n\bar{T}s) + e_q(n\bar{T}s)$$

Substituting the value of $e_q(n\bar{T}s)$, we get

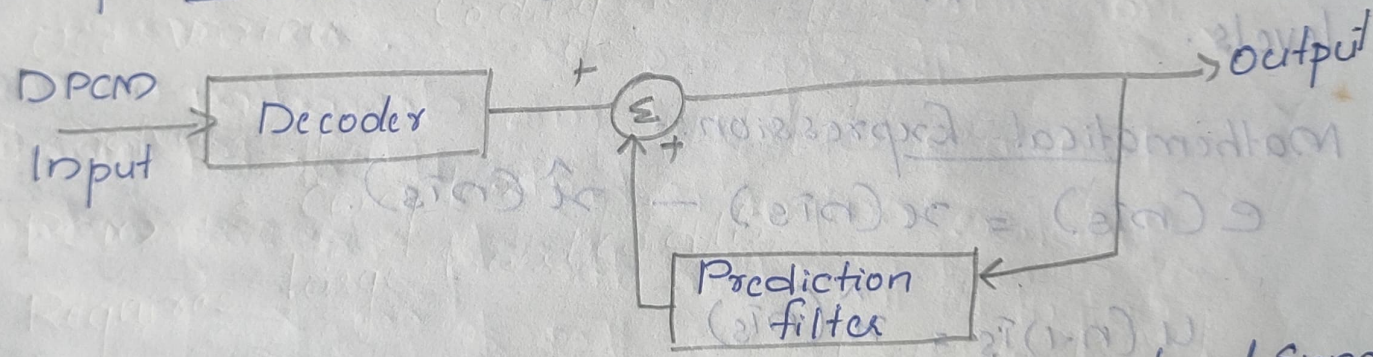
$$x_q(n\bar{T}s) = \hat{x}(n\bar{T}s) - \hat{x}(n\bar{T}s).$$

$$e(n\bar{T}s) + \hat{x}(n\bar{T}s) = x(n\bar{T}s)$$

The value of $e(n\bar{T}s) + \hat{x}(n\bar{T}s)$

$$x_q(n\bar{T}s) = x(n\bar{T}s) + q(n\bar{T}s).$$

Reception of DPCM Signal:-



Decoder first reconstructs the quantised signal from incoming binary signal. The prediction filter output and quantised error signals are summed upto give quantised version of orginal signal. Thus the signal at the receiver differs from actual signal by $q(n\bar{T}s)$ which is introduced permanently in the reconstructed signal.

Delta Modulation:-

Reason:-

PCM transmits all the bits which are used to code a sample. Hence signalling rate and transmission channel bandwidth are quite large in PCM.

Working Principle:-

Delta modulation transmits only one bit per sample. Here the present value is compared with the previous sample value and the result whether the amplitude is increased or decreased is transmitted Input signal $x(t)$ is approximated to step signal by the delta modulator. This step size is kept fixed. The difference between the input signal $x(t)$ and staircase is approximated signal to confined to two levels.

Mathematical Expression:-

$$e(n\bar{T}s) = x(n\bar{T}s) - \hat{x}(n\bar{T}s).$$

$$u(n-1)\bar{T}s = \hat{x}(n\bar{T}s)$$

$$b(n\bar{T}s) = \Delta \, Sgn\,(e(n\bar{T}s))$$

$$b(n\bar{T}s) = \begin{cases} +\Delta & \text{if } x(n\bar{T}s) \geq \hat{x}(n\bar{T}s) \\ -\Delta & \text{if } x(n\bar{T}s) \leq \hat{x}(x\bar{T}s) \end{cases}$$

Transmitter:-

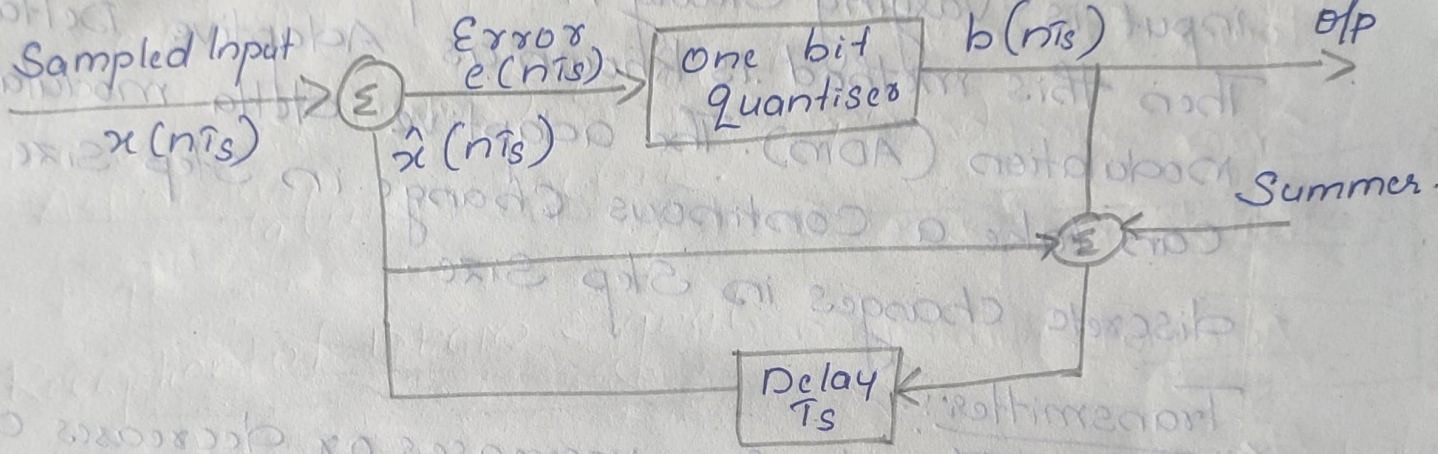
Summer add quantiser output with previous sample approximation

$$u(n\bar{T}s) = u(n\bar{T}s - \bar{T}s) + \pm [\Delta].$$

If Step size is $+\Delta$ the binary 1 is transmitted and if it is $-\Delta$, then binary 0 is transmitted.

Receiver Part :-

The accumulator and LPF are used. Accumulator generates the staircase approximated signal output and is delayed by one sampling period Ts



Advantages :-

- Signalling rate and transmission channel band width is quite
- Much Simple

Drawbacks :-

- Slope overload distortion
- Granular Noise.

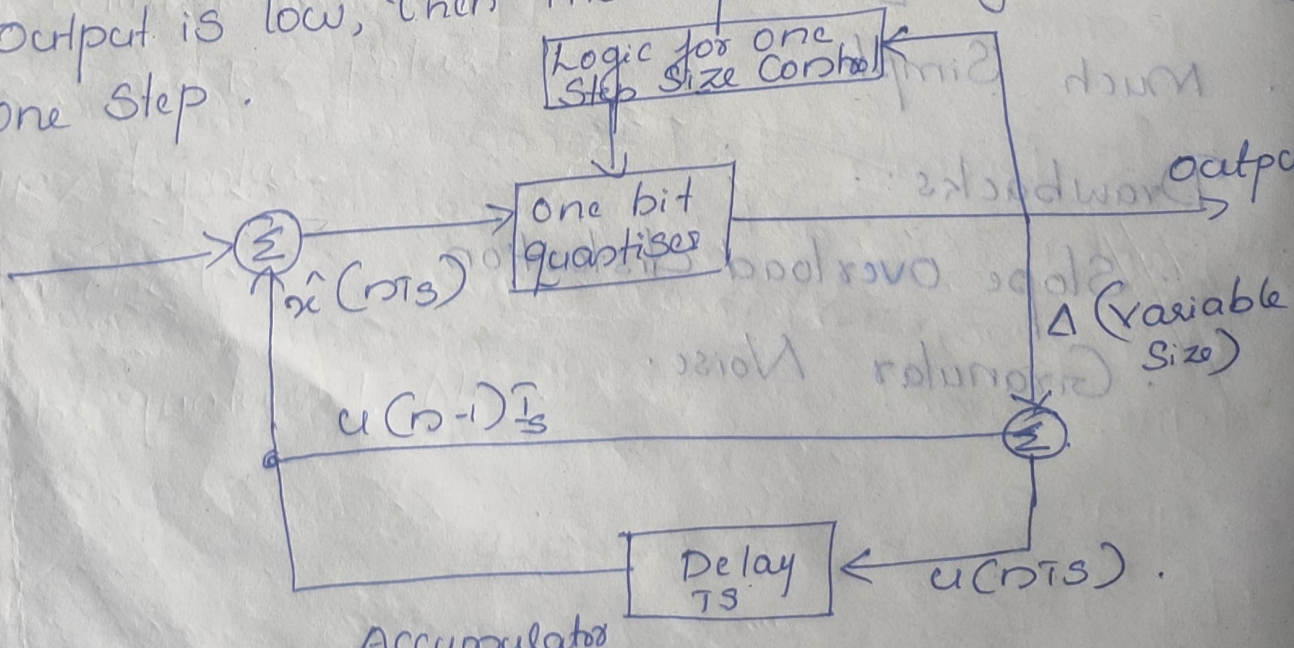
Adaptive Delta Modulation:-

Reason to use Adaptive Delta Modulation:-

To overcome the quantization errors due to slope overload and granular noise, the step size Δ is made adaptive to variations in the input signal $x(t)$. Particularly in the step segment of the signal $x(t)$, step size is increased. Also the input is varying slowly, the step size is reduced. Then this method is known as Adaptive Delta Modulation (ADM). The adaptive delta modulators can take a continous change in step size or discrete changes in step size.

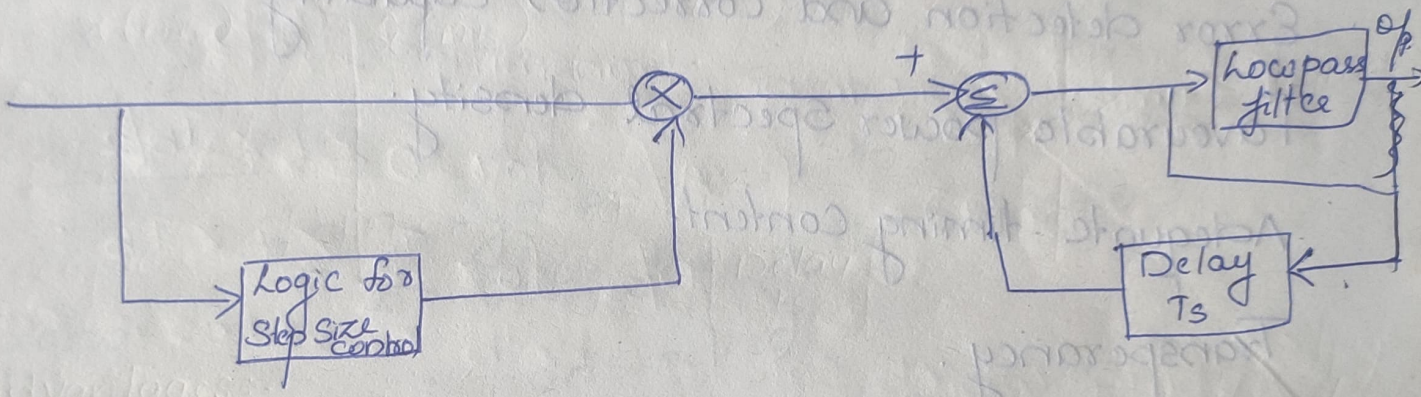
Transmitter:-

The step size increases or decreases according to a specified rule depending on one bit quantiser output is high. If one bit quantiser output is low, then the step size may reduced by one step.



Receiver Part:-

In the receiver of adaptive delta modulator there are two portions. The first portion produces the step size from each incoming bit. Exactly the same process is followed as that in transmitter. The previous input and present input decides the step size. It is then applied to an accumulator which builds up staircase waveform



Advantages:-

Adaptive delta modulation has certain advantages

(i) the signal to noise ratio become better than ordinary delta modulation.

(ii) because of the variable step size, the dynamic range of ADM is wider than simple DM

(iii) Utilization of bandwidth is better than delta modulation.

8/7/16

Line Coding Schemes:-

The digital data can be transmitted by various transmission or line codes such as ON-off, polar, bipolar and so on. This is called line coding.

Properties:-

Transmission Bandwidth.

Power Efficency.

Error detection and Correction capability.

Favourable power spectral density.

Adequate timing content.

Transperancy.

Various Schemes:-

Non Return to Zero (NRZ) and Return to Zero (RZ)

NRZ and RZ polar

NRZ bipolar

Manchester.

Polar quaternary NRZ

Unipolar RZ and NRZ:-

In unipolar format, there will be single polarity. The waveform can be +5 or +12 V whereas high. This waveform is simple ON-OFF.

If symbol '1' is transmitted

$$x(t) = \begin{cases} A & \text{for } 0 \le t \le T_b/2 \\ 0 & \text{for } T_b/2 \le t < T_b \end{cases}$$

Unipolar NRZ:-

When symbol '1' is to be transmitted Signal has 'A' volts for full duration.

If symbol '1' is transmitted

$$x(t) = A \quad 0 \le t \le T_b$$

If symbol '0' is transmitted

$$x(t) = 0 \quad 0 \le t \le T_b$$

Polar RZ and NRZ:-

In polar RZ '1' is represented by positive voltage and '0' is represented by -ve voltage.

For symbol '1'

$$x(t) = \begin{cases} +A/2 & \text{for } 0 \le t < T_b/2 \\ 0 & \text{for } T_b/2 \le t < T_b \end{cases}$$

Bipolar:

In this format, the successive '1's are represented by pulses with alternate polarity and 0's with pulses.

Manchester:

If Symbol '1' is to be transmitted, then

$$x(t) = \begin{cases} A/2 & \text{for } 0 \le t < T_b/2 \\ -A/2 & \text{for } T_b/2 \le t < T_b \end{cases}$$

If Symbol '0' is to be transmitted then

$$x(t) = \begin{cases} -A/2 & \text{for } 0 \le t < T_b/2 \\ A/2 & \text{for } \frac{T_b}{2} \le t < T_b \end{cases}$$

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30/01/26 Module-II (13 hours)

Shaping :- Nyquist Criterion for Zero ISI :-

In Communication System, when the data is being transmitted in the form of pulses the output produced at the receiver due to other bits or symbol interferes with the output produced by desired bit. This is known as intersymbol interferance.

Factors responsible for ISI :-
- Imperfections in overall frequency.
- when short duration pulse are diffrentially attenuated and diffrently delayed.

Signal Design for Zero ISI :-

The pulse amplitude $a_k = \begin{cases} +1 & \text{if } b_k \text{ is symbol 1} \\ -1 & \text{if } b_k \text{ is symbol 0} \end{cases}$

The transmitted signal will be

$$s(t) = \sum_k a_k g(t - kT_b).$$

$$y(t) = \sum_k a_k g(t - kT_b)$$

Scaled pulse $p(t)$ is obtained by a double convolution involving the impulse response $g(t)$ of transmit filter

$$p(t) = g(t) * h(t) * c(t).$$

$$y(t_i) = \sum_{k=-\infty}^{\infty} a_k p\left[(i-k)T_b\right].$$

$$= a_i + \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p\left[(i-k)T_b\right].$$

represents the contribution of the i^{th} transmitted bits

$$y(t_i) = a_i$$

The signal $a_k p(iT_b - kT_b)$, must be zero for all k except k=1

$$p(iT_b - kT_b) = \begin{cases} 1 & \text{for } i=k \\ 0 & \text{for } i \neq k \end{cases}$$

A pulse which satisfies both condition is called Nyquist pulse and the condition is called Nyquist's criterion for distortionless binary base band data transmission.

$$P_S(f) = R_b \sum_{n=-\infty}^{\infty} p(f - nR_b).$$

$R_b = 1/T_b$ is the bit rate/s.

$$P_S(f) = \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} p\left[(mT_b)\delta(t-mT_b)\right] e^{-j2\pi ft} \cdot dt.$$

$$P_S(f) = p(0) \int_{-\infty}^{\infty} \delta(t)\, e^{-j2\pi ft}\, dt.$$

$$\sum_{n=-\infty}^{\infty} p(f - nR_b) = T_b.$$

This is the Nyquist criterion for distortionless baseband transmission

Signalling with duobinary pulses:-

Consider a binary input sequence $\{b_k\}$ consisting of uncorrelated digits having duration T_b seconds. Let the symbol 1 be represented by a pulse of amplitude +1 V and 0 is -1 V. When this sequence is applied to a duobinary encoder, it is converted into three level output. Inorder to produce this transformation, the digit c_k of duobinary coder output may be expressed as sum of the present binary digit and its previous value

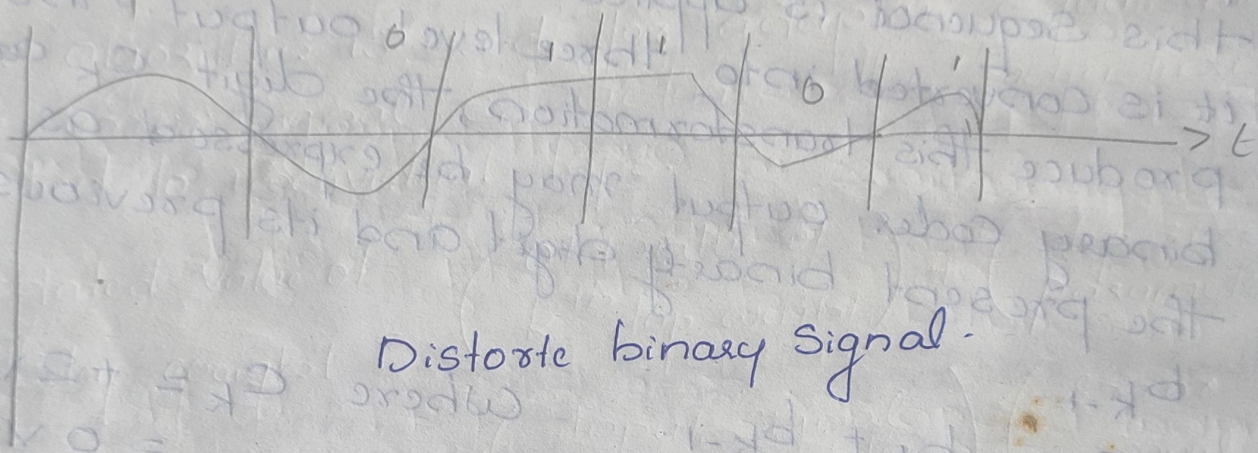
$$c_k = b_k + b_{k-1}$$

where $c_k = +2$ V $b_k = b_{k-1} = 1$
$= 0$ V $b_k \neq b_{k-1}$
$= -2$ V $b_k = b_{k-1} = 0$

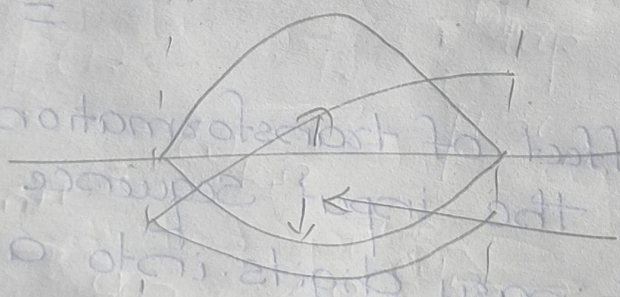
The effect of transformation described is that changes the input sequence $\{b_k\}$ of uncorrelated binary digits into a sequence $\{c_k\}$ of correlated digits. This correlation between adjacent transmitted levels introduce ISI. This effect can be compensated at the receiver.

Concept of Eye Pattern:-

Eye pattern is a pattern displayed on the Screen of a CRO. The Shape of this pattern resembles with shape of human eye. Therefore it is called as eye pattern. It is a practical method to study isi and its effects on a PCM. The interior region is called eye opening.



Distorte binary signal.



Eye pattern on CRO.

Information obtained from Eye pattern:-

- The width of eye opening defines the time interval.
- Sensitivity of System can be measured.
- Height Specified the margin over noise.

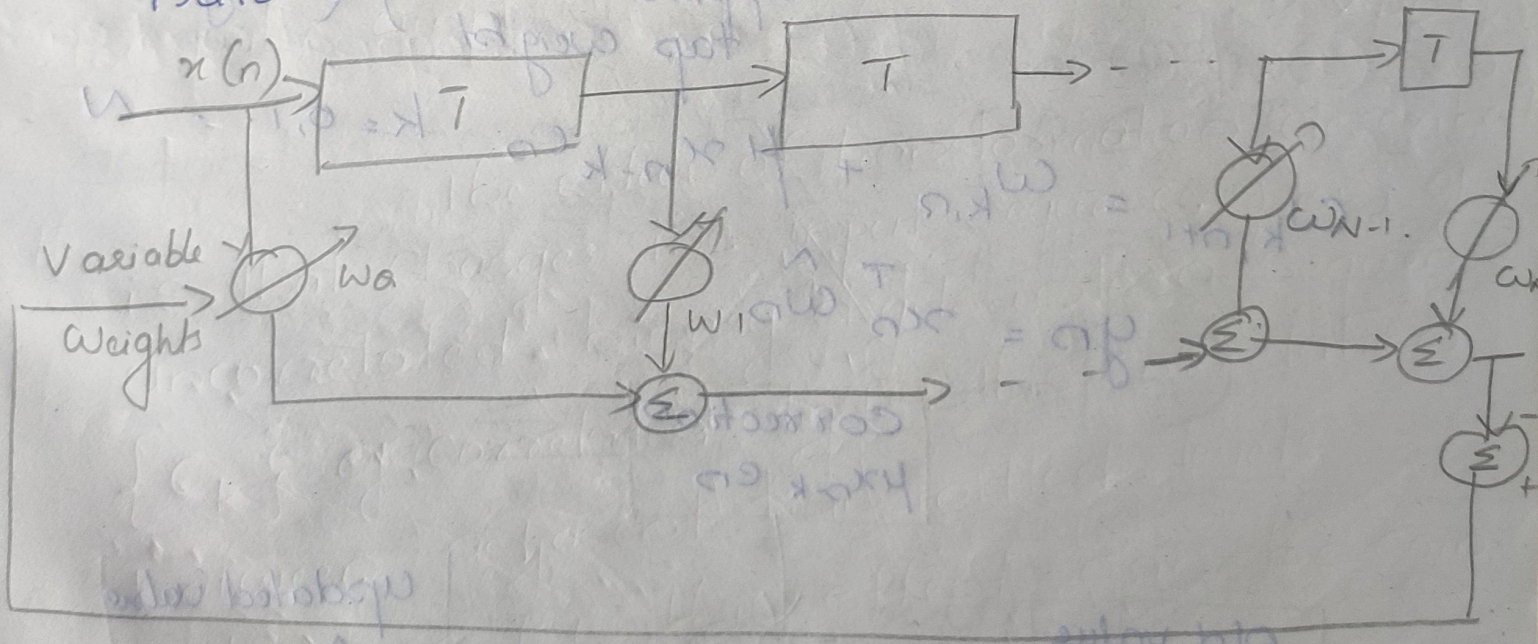
. when effect of ISI is severe, the eye is completely closed and it is impossible to avoid errors.

Equaliser:

Channels can be characterised by transfer function $H_c(f) = |H_c(f)| e^{-j\theta_c(f)}$

$h_c(t)$ and $H_c(f)$ are Fourier Transform pairs

Inorder to achieve ideal transmission characteristics over a channel $|H_c(f)|$ should be constant. If $H_c(f)$ is not constant the effect will be amplitude distortion. Equalisation refers to signal processing that is designed to eliminate or reduce ISI.



$$x_n = x(nT).$$

$$y_n = y(nT)$$

Output $y_n = \sum_{k=0}^{N} w_k x_{n-k}$

where w_k is the weight at k^{th} tap and N+1 is total number of taps.

Let a_n denote the desired response defined as polar representation of the n^{th} transmitted binary symbol.

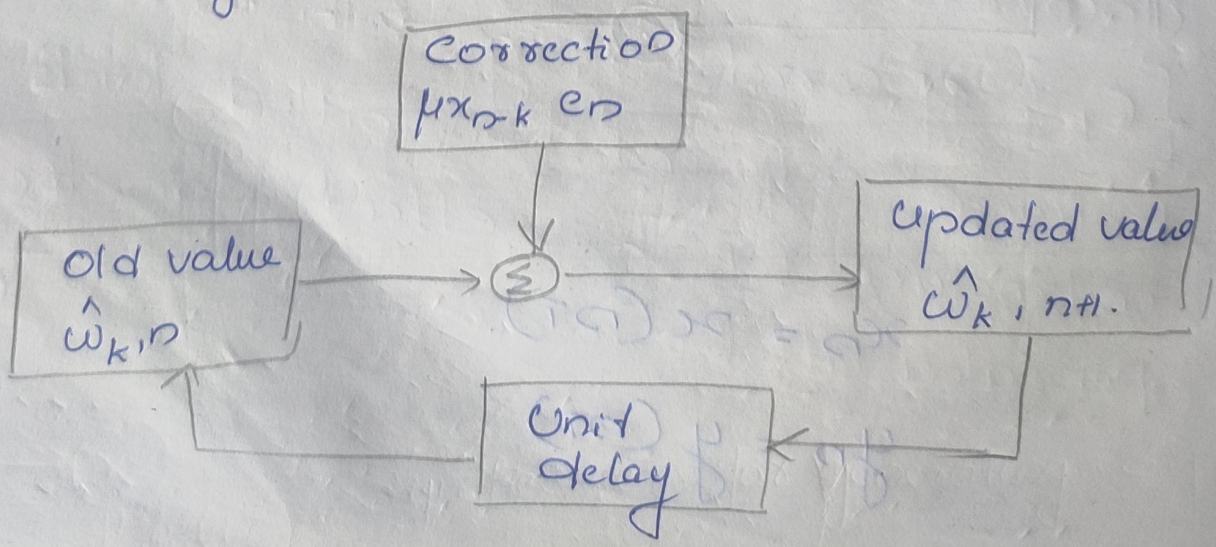
$$e_n = a_n - y_n$$

LMS algorithm is used for adaptive equalisation. The error signal e_n actuates the adjustments applied to individual taps.

Updated value of k^{th} tap weight = (Old value of k^{th} tap weight) + (Step size parameter). (Input signal applied to k^{th} tap weight)(Error Signal)

$$w_{k,n+1} = w_{k,n} + \mu x_{n-k} e_n \qquad k = 0, 1 \ldots N$$

$$y_n = x_n^T \hat{w}_n$$



Signal Space Concepts:-

A 3 dimensional vector is an entity of signal space. Consider the most basic form of a digital communication system

$$P_i = P(m_i)$$

$$= \frac{1}{M} \quad \text{for } i = 1, 2, \ldots M$$

Source emits symbols every time ie every T seconds and the symbol will be $m_1, m_2 \ldots m_n$.

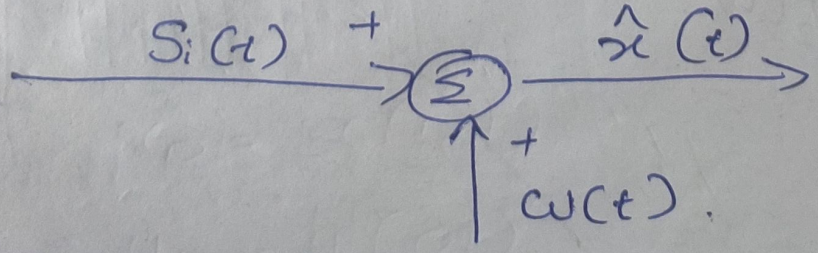
The transmitter takes the symbol and codes into distinct signal $S_i(t)$ suitable for transmission over channel. The signal $S_i(t)$ occupies full duration T alloted to symbol m_i. $S_i(t)$ will be a real valued energy signal

$$E_i = \int_0^T S_i^2(t)\, dt \qquad i = 1, 2 \ldots M$$

Channel is an additive white Gaussian

$$x(t) = S_i(t) + \omega(t) \qquad \begin{cases} 0 \le t \le T \\ i = 1, 2 \ldots M \end{cases}$$

$$P_e = \sum_{i=1}^{M} P_i \, P(\hat{m} \neq m_i | m_i)$$



AWGN model of a channel.

Geometric Representation of Signals:-

It is used to represent any set of M energy signals $\{s_i(t)\}$ as linear combinations of N orthonormal basis functions, where $N \leq M$.

$$s_i(t) = \sum_{j=1}^{N} s_{ij} \phi_j(t) \quad \begin{cases} 0 \leq t \leq T \\ i = 1, 2 \ldots M. \end{cases}$$

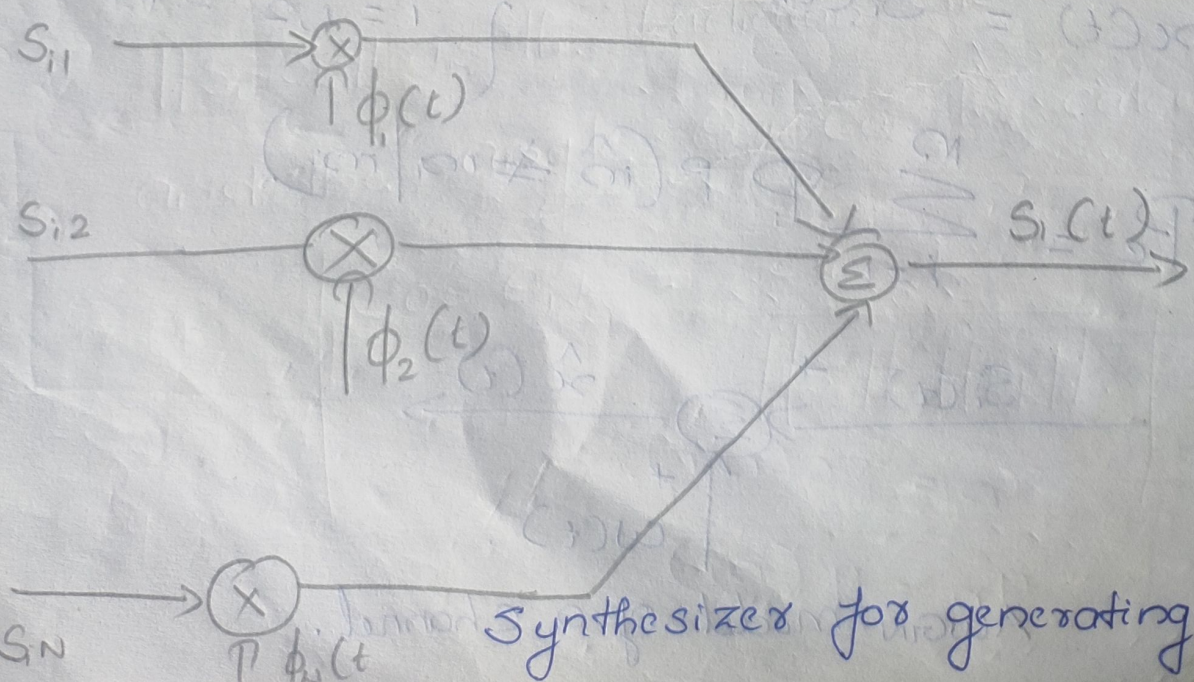
$$s_{ij} = \int_0^T s_i(t) \phi_j(t) \, dt.$$

The real valued basis functions $\phi_1(t), \phi_2(t) \ldots \phi_N(t)$ are orthonormal by which

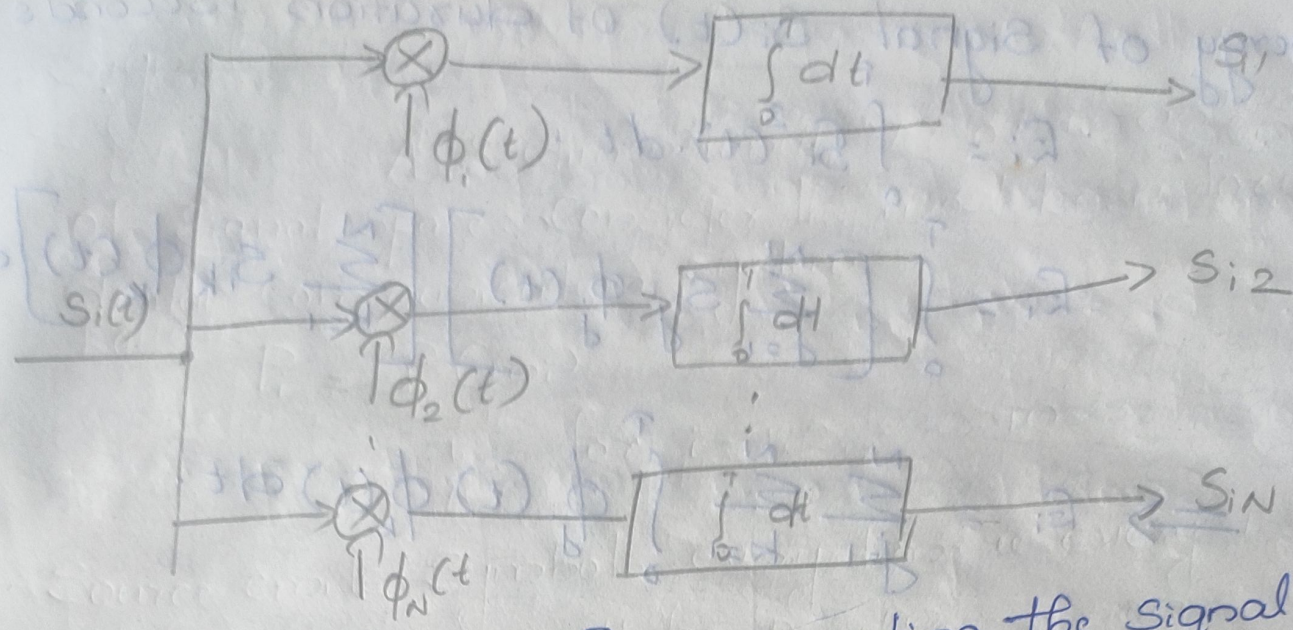
$$\int_0^T \phi_i(t) \phi_j(t) \, dt = \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

where δ_{ij} is Kronecker delta.

1. Each function should be normalised.
2. $\phi_1(t), \phi_2(t) \ldots, \phi_N(t)$ are orthogonal w.r.t each other.



Synthesizer for generating signal $s_i(t)$



Analyzer for generating the signal vector $\{S_i\}$.

L^2 Space distance :-

Accordingly, we may state that each signal in the set $\{S_i(t)\}$ is completely determined by vector of its coiefficent

$$S_i = \begin{bmatrix} S_{i1} \\ S_{i2} \\ \vdots \\ S_{iN} \end{bmatrix} \qquad i = 1, 2 \ldots N.$$

The vector S_i is called a signal vector. It can be noted in two or three dimension to 'n' dimension. we can define in two ways.

- Length vector.
- Angles vector.

$$\|S_i\|^2 = S_i^T S_i$$

$$= \sum_{j=1}^{N} S_{ij}^2 \qquad i = 1, 2 \ldots M.$$

Energy of Signal $S_i(t)$ of duration T seconds is

$$E_i = \int_0^T S_i^2(t)\,dt.$$

$$E_i = \int_0^T \left[\sum_{j=1}^{N} S_{ij}\,\phi_j(t)\right]\left[\sum_{k=1}^{N} S_{ik}\,\phi_k(t)\right]dt$$

$$\Longrightarrow E_i = \sum_{j=1}^{N}\sum_{k=1}^{N} \int_0^T \phi_j(t)\,\phi_k(t)\,dt.$$

$$E_i = \sum_{j=1}^{N} S_{ij}^2 = \|S_i\|^2$$

In case of pair of signals $S_i(t)$ and $S_k(t)$ over the interval $[0, T]$

$$\|S_i - S_k\|^2 = \sum_{j=1}^{N} (S_{ij} - S_{kj})^2.$$

$$= \int_0^T [S_i(t) - S_k(t)]^2\,dt.$$

$\|S_i - S_k\|$ is the Euclidean distance.

and the angle θ_{ik} can be calculated as

$$\boxed{\cos\theta_{ik} = \frac{S_i^T S_k}{\|S_i\|\,\|S_k\|}.}$$

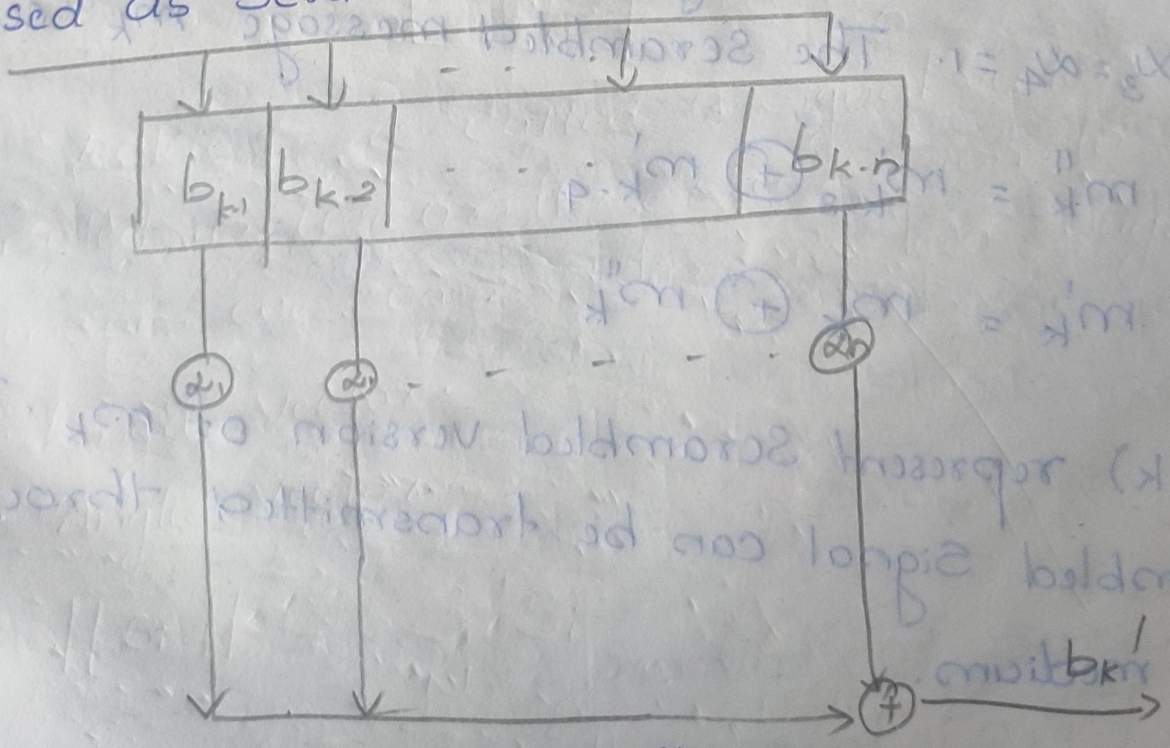
Scrambling And Descrambling

In communication it is essential to transmit the message in such a form that only the desired receiver should be able to understand it.

Scrambling is a coding operation applied to the message at the transmitter. The process of scrambling will randomise the bit stream eliminating long strings of 0's and 1's which might affect receiver synchronisation. The scrambling process also eliminates most periodic bit patterns which can produce un desirable discrete frequency components in power spectrum. At receiver end the signal is descrambled.

Tapped Shift Register:-

A tapped shift register is commonly used as Scrambler.



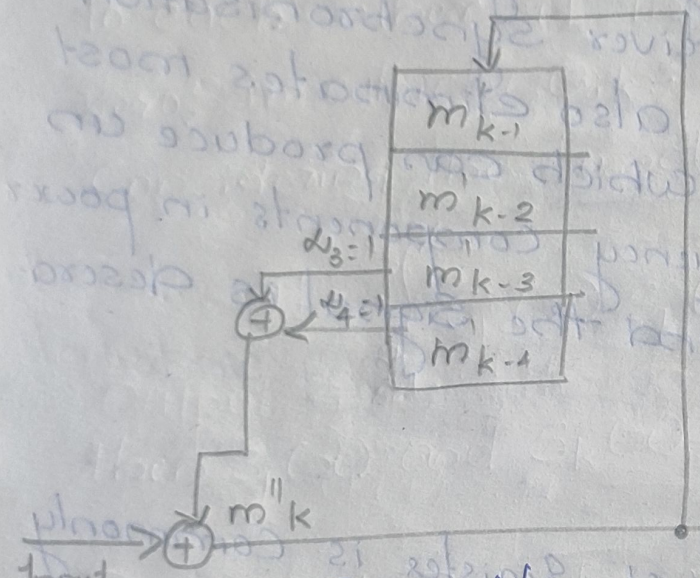
Tapped Shift Register.

b_k is the binary input sequence. Successive bits from b_k enter the register and shift from one stage to next for each clock pulse

$$\therefore b_k' = \alpha_1 b_{k-1} \oplus \alpha_2 b_{k-2} \oplus \cdots \oplus \alpha_n k-n$$

Scrambler:-

Scramblers use n stage shift registers.



Scrambler

It uses a 4 stage shift register with $\alpha_1 = \alpha_2 = 0$ and $\alpha_3 = \alpha_4 = 1$. The scrambled message m_k' will be

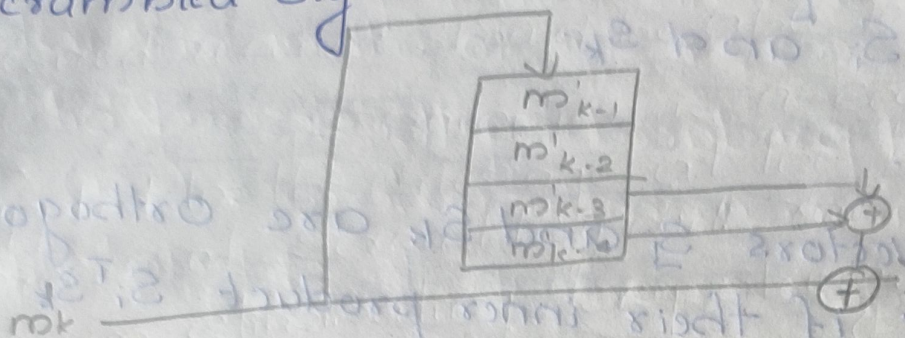
$$m_k'' = m_{k-3}' \oplus m_{k-4}'$$

$$m_k' = m_k \oplus m_k''$$

$m'(k)$ represent Scrambled version of m_k. The Scrambled signal can be transmitted through any medium.

Descrambling :-

is used to recover orginal message from scrambled signal.



The unscrambler structure is exactly reverse as compared to scrambler. By adding the scrambled Signal m_k' it is possible to obtain orginal signal m_k.

Disadvantages :-

- Error keep propagating untill the register is full of correct bits.
- Loss of Synchronisation

Inner Product :-

In case of pair of symbols $S_i(t)$ and $S_k(t)$ represented by signal vectors S_i and S_k respectievly

$$\int_0^T S_i(t) S_k(t) \cdot dt = S_i^T S_k$$

This is the inner product of signals $S_i(t)$ and $S_k(t)$ over interval $[0, T]$

using their time domain representation equal to the inner product of their respective vector representation which is equal to the inner product of their vector S_i and S_k.

Orthogonality:

Two vectors S_j and S_k are orthogonal to each other if their inner product $S_i^T S_k$ is zero, in which $\theta_{ik} = 90^\circ$.

Considering the functions $S_j(x)$ and $S_k(x)$ if

$$\int_{x_1}^{x_2} S_j(x)\, S_k(x)\, dx = 0$$

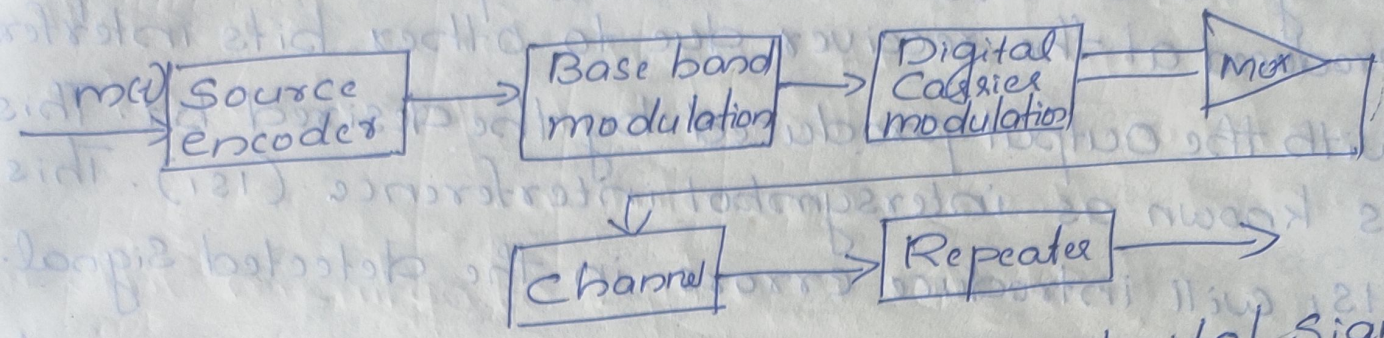
then $S_j(x)$ and $S_k(x)$ are orthogonal.

Base Band Data Transmission:

The discrete signal cannot be transmitted as such over the channel. If such a signal is transmitted over a bandlimited channel, then the signal gets dispersed. This causes the pulses to overlap and cause distortion. In order to avoid this, we should not transmit the discrete signal as it is on the transmission medium. Infact first data is converted into a suitable format and then transmitted over a communication channel.

This formats are called line codes.

Block Diagram:



Source encoder converts digital signal into sequence of digits. This data is converted into formats called line codes. Various line codes are

- NRZ
- RZ
- Manchester.

The capacity of a physical channel such as optical, co-axial etc can be used. Regenerative repeaters are placed in regular space intervals along transmission line. Further repeate create an exact duplicate of incoming data by identifying it by noise, reconstructing and retransmitting by desired information. A repeater should be precise over the link such that the signal reaches it before the noise has induced an error in any of the bits.

Inter Symbol Interferance (ISI):-

In Communication System, when data is being transmitted in the form of pulses the output produced at the receiver due to other bits interferes with the output produced by the desired bit. This is known as intersymbol interference (ISI). This ISI will introduce errors in the detected signal.

The Sequence $x(t)$ is applied to a pulse generator

$$x(t) = \sum_{k=-\infty}^{\infty} a_k V(t - kT_b)$$

Output after filtering will be

$$y(t) = \mu \sum_{k=-\infty}^{\infty} a_k p(t - kT_b) + n(t).$$

The receiving filter output $y(t)$ is Sampled $t_i = iT_b$

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} a_k p(iT_b - kT_b) + n(t_i).$$

$$y(t_i) = \mu a_i + \mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} a_k p(iT_b - kT_b) + n(t_i)$$

- μa_i is produced by the i^{th} transmitted bit
- 2nd term represent ISI

Factors Responsible for ISI:-

ISI arises due to imperfections in the overall frequency response of a system. When a short pulse of duration T_b seconds is transmitted through a bandlimited system, then the frequency components contained in the input pulse are differentially attenuated and more importantly diffrentially delayed.

Effect of ISI:-

- In the absence of ISI and noise, the transmitted bit can be decoded correctly at the receiver
- Presence of ISI will introduce errors in the decision at receiver output.

Remedy:

- Function which produces a zero intersymbol interferance is a sinc function.
- Fourier Transform of a sinc pulse is a rectangular function.
- Nyquist criterion has to be followed.

Matched Filter Receiver:-

A matched filter is a linear filter designed to provide maximum signal to noise power ratio at its output for a transmitted waveform. The mathematical operation of a matched filter is convolution.

This filter is a type of optimum filter in which the noise is white Gaussian noise. The power spectral density of white Gaussian noise is

$$S_{ni}(f) = \frac{N_0}{2}$$

Impulse Response of matched Filter:-

The transfer function of optimum filter is

$$H(f) = k \cdot \frac{X^*(f)}{S_{ni}(f)} e^{-j2\pi fT}$$

Substitute $S_{ni}(f) = \frac{N_0}{2}$ i.e power spectral density of white noise.

Then $$H(f) = k \cdot \frac{X^*(f)}{\frac{N_0}{2}} e^{-j2\pi fT}$$

or $$H(f) = \frac{2k}{N_0} X^*(f) e^{-j2\pi fT}$$

From the property of fourier transform

$$X^*(f) = X(-f)$$

$$\therefore H(f) = \frac{2k}{N_0} X(-f) e^{-j2\pi fT}$$

The impulse response of a matched filter can be evaluated by taking inverse fourier transform

$$h(t) = IFT[H(f)] = IFT\left[\frac{2k}{N_0} X(-f) e^{-j2\pi fT}\right]$$

$$h(t) = IFT\left[\frac{2k}{N_0} X(-f) e^{-j2\pi fT}\right]$$

$$FT[x(-t)] = X(-f).$$

$$FT[x(T-t)] = X(-f) e^{-j2\pi fT}$$

$$h(t) = \frac{2k}{N_0} x(T-t).$$

Consider $x(t) = x_1(t) - x_2(t)$.

$$h(t) = \frac{2k}{N_0}\left[x_1(T-t) - x_2(T-t)\right]$$

This is the required impulse response of matched filter.

Calculation of probability of Error (P_e) for the matched filter

Error probability of optimum filter is

$$P_e = \frac{1}{2}\operatorname{erfc}\left[\frac{x_{o1}(T) - x_{o2}(T)}{2\sqrt{2}\,\sigma}\right]$$

$$\left[x_{o1}(T) - x_{o2}(T)\right]^2_{max} = \int_{-\infty}^{\infty} \frac{|X(f)|^2}{S_{ni}(f)}\,df$$

Power spectral density $= S_{ni}(f) = \frac{N_0}{2}$

$$\left[\frac{x_{o1}(T) - x_{o2}(T)}{\sigma}\right]^2_{max} = \int_{-\infty}^{\infty} \frac{|X(f)|^2}{\frac{N_0}{2}}\,df$$

Verified V. Pal 8/7/16

$$= \frac{2}{N_0}\int_{-\infty}^{\infty} |X(f)|^2\,df$$

Parseval's power theorem

The normalised energy of a periodic waveform $V(t)$ in a period T_0 is

$$E = \int_{-T_0/2}^{T_0/2} [V(t)]^2\,dt$$

$$E = T_0 s = T_0 \sum_{n=-\infty}^{\infty} V_n V_n^*$$

$$V_n V_n^* = \frac{V_n}{\Delta f}\,\frac{V_n^*}{\Delta f}\,(\Delta f)^2$$

$$E = \int_{-\infty}^{\infty} [V(t)]^2\,dt$$

Gram Schmidt Orthogonalisation Procedure:-

Suppose we have a set of M energy signals denoted by $s_1(t), s_2(t) \ldots s_M(t)$. Starting with $s_1(t)$, the first basis function is

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}$$

where E_1 is the energy of signal $s_1(t)$.

$$s_1(t) = \sqrt{E_1}\,\phi_1(t)$$

$$= s_{11}\,\phi_1(t)$$

where $s_{11} = \sqrt{E_1}$ and $\phi_1(t)$ is unit energy.

Signal $s_2(t)$ coiefficent

$$s_{21} = \int_0^T s_2(t)\,\phi_1(t)\,dt.$$

We introduce an intermediate function

$$g_2(t) = s_2(t) - s_{21}\,\phi_1(t)$$

which is orthogonal to $\phi_1(t)$ over the interval $0 \le t \le T$

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^2(t)\,dt}}$$

Simplifying

$$\phi_2(t) = \frac{s_2(t) - s_{21}\phi_1(t)}{\sqrt{\mathcal{E}_2 - s_{21}^2}}$$

where $\mathcal{E}_2$ is energy of signal.

$$\int_0^T \phi_2^2(t)\,dt = 1.$$

$$\int_0^T \phi_1(t)\,\phi_2(t)\,dt = 0.$$

Continuing we may define

$$g_i(t) = s_i(t) - \sum_{j=1}^{i-1} s_{ij}\,\phi_j(t).$$

Given $g_i(t)$ we define basis function

$$\phi_i(t) = \frac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)\,dt}} \qquad i = 1, 2 \cdots N.$$

The signals $s_1(t), s_2(t) \cdots s_N(t)$ form linearly independent set

Signals $s_1(t), s_2(t) \cdots s_N t$ are not linearly independent.

The form of basis functions $\phi_1(t), \phi_2(t) \ldots \phi_N(t)$ has not been specified.

The expansion of signal $S_i(t)$ interms of finite number of terms is not an approximation wherein only first N terms are significant.